All of us have seen pictures of the Pyramids of Egypt. These amazing structures were built very carefully. Measurements had to be made that were very precise. The ancient Egyptians used an elaborate system of fractions that allowed them to make highly accurate measurements. As you master the topics of this module, you will master the basic skills used by the designers of the Pyramids of Egypt.

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Fractions

Section 1 Understanding Fractions

Student Learning Objectives

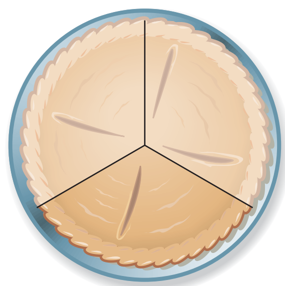
After studying this section, you will be able to:

1. **Use a fraction to represent part of a whole.**
2. **Draw a sketch to illustrate a fraction.**
3. **Use fractions to represent real-life situations.**

1 Using a Fraction to Represent Part of a Whole

In this module we will study a fractional part of a whole number. One way to represent parts of a whole is with **fractions.** The word *fraction* (like the word *fracture*) suggests that something is being broken. In mathematics, fractions represent the part that is "broken off" from a whole. The whole can be a single object (like a whole pie) or a group (the employees of a company). Here are some examples.

Single object



$$\frac{1}{3}$$

The whole is the pie on the left. The fraction $\frac{1}{3}$ represents the shaded part of the pie, 1 of 3 pieces. $\frac{1}{3}$ is read "one-third."

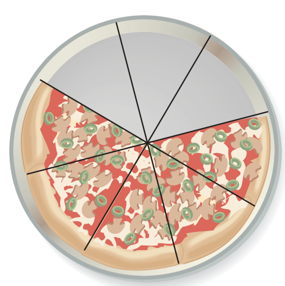
A group: ACE company employs 150 men, 200 women.

$$\frac{150}{350}$$

The whole is the company of 350 people (150 men plus 200 women). The fraction $\frac{150}{350}$ represents that part of the company consisting of men.

Recipe: Applesauce
4 apples
1/2 cup sugar
1 teaspoon cinnamon

The whole is 1 whole cup of sugar. This recipe calls for $\frac{1}{2}$ cup of sugar. Notice that in many real-life situations $\frac{1}{2}$ is written as 1/2.



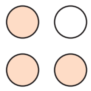
When we say "$\frac{3}{8}$ of a pizza has been eaten," we mean 3 of 8 equal parts of a pizza have been eaten. (See the figure.) When we write the fraction $\frac{3}{8}$, the number on the top, 3, is the **numerator,** and the number on the bottom, 8, is the **denominator.**

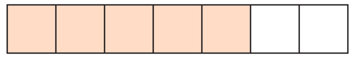
$$\begin{array}{r l} \text{The numerator specifies how many parts} \rightarrow & 3 \\ \hline \text{The denominator specifies the total number of parts} \rightarrow & 8 \end{array}$$

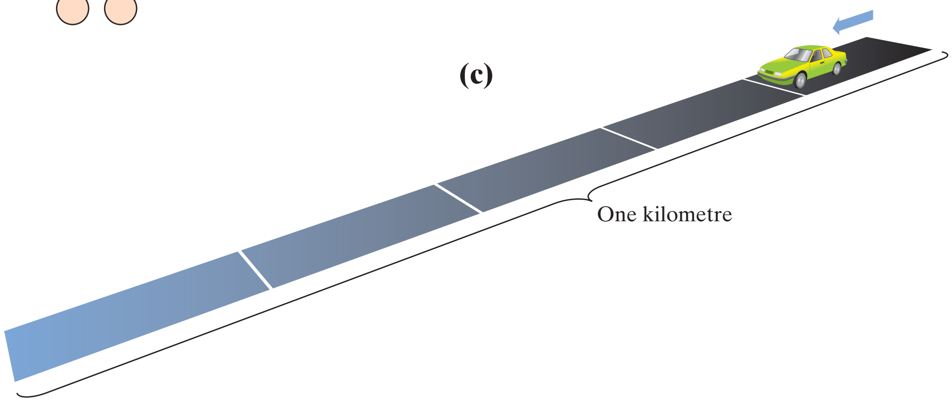
When we say, "$\frac{2}{3}$ of the marbles are red," we mean 2 marbles out of a total of 3 are red marbles.

$$\begin{array}{r l} \text{Part we are interested in} \rightarrow & 2 \ \text{numerator} \\ \hline \text{Total number in the group} \rightarrow & 3 \ \text{denominator} \end{array}$$

EXAMPLE 1 Use a fraction to represent the shaded or completed part of the whole shown.

(a) 

(b) 

(c) 

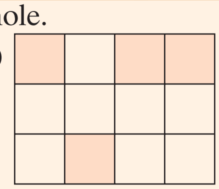
Solution

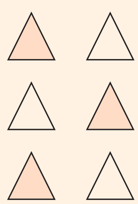
(a) Three out of four circles are shaded. The fraction is $\frac{3}{4}$.

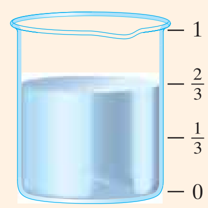
(b) Five out of seven equal parts are shaded. The fraction is $\frac{5}{7}$.

(c) The kilometre is divided into five equal parts. The car has travelled 1 part out of 5 of the one-kilometre distance. The fraction is $\frac{1}{5}$.

Practice Problem 1 Use a fraction to represent the shaded part of the whole.

(a) 

(b) 

(c) 

***NOTE TO STUDENT:** Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.*

We can also think of a fraction as a division problem.

$$\frac{1}{3} = 1 \div 3 \qquad \text{and} \qquad 1 \div 3 = \frac{1}{3}$$

The division way of looking at fractions asks the question:

What is the result of dividing one whole into three equal parts?

Thus we can say the fraction $\frac{a}{b}$ means the same as $a \div b$. However, special care must be taken with the number 0.

Suppose that we had four equal parts and we wanted to take none of them. We would want $\frac{0}{4}$ of the parts. Since $\frac{0}{4} = 0 \div 4 = 0$, we see that $\frac{0}{4} = 0$. Any fraction with a 0 numerator equals zero.

$$\frac{0}{8} = 0 \qquad \frac{0}{5} = 0 \qquad \frac{0}{13} = 0$$

What happens when zero is in the denominator? $\frac{4}{0}$ means 4 out of 0 parts. Taking 4 out of 0 does not make sense. We say $\frac{4}{0}$ is **undefined.**

$$\frac{3}{0}, \ \frac{7}{0}, \ \frac{4}{0} \quad \text{are } \textbf{undefined.}$$

We cannot have a fraction with 0 in the denominator. Since $\frac{4}{0} = 4 \div 0$, we say division by zero is *undefined.* We cannot divide by 0.

2 Drawing a Sketch to Illustrate a Fraction

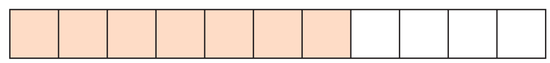
Drawing a sketch of a mathematical situation is a powerful problem-solving technique. The picture often reveals information not always apparent in the words.

EXAMPLE 2 Draw a sketch to illustrate.

(a) $\dfrac{7}{11}$ of an object **(b)** $\dfrac{2}{9}$ of a group

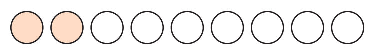
Solution

(a) The easiest figure to draw is a rectangular bar.



We divide the bar into 11 equal parts. We then shade in 7 parts to show $\dfrac{7}{11}$.

(b) We draw 9 circles of equal size to represent a group of 9.



We shade in 2 of the 9 circles to show $\dfrac{2}{9}$.

NOTE TO STUDENT: ***Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.***

Practice Problem 2 Draw a sketch to illustrate.

(a) $\dfrac{4}{5}$ of an object **(b)** $\dfrac{3}{7}$ of a group

Recall these facts about division problems involving the number 1 and the number 0.

DIVISION INVOLVING THE NUMBER 1 AND THE NUMBER 0

1. Any nonzero number divided by itself is 1.

$$\frac{7}{7} = 1$$

2. Any number divided by 1 remains unchanged. $\dfrac{29}{1} = 29$

3. Zero may be divided by any nonzero number; the result is always zero.

$$\frac{0}{4} = 0$$

4. Division by zero is undefined. $\dfrac{3}{0}$ is undefined

3 Using Fractions to Represent Real-Life Situations

Many real-life situations can be described using fractions.

EXAMPLE 3 Use a fraction to describe each situation.

(a) A baseball player gets a hit 5 out of 12 times at bat.

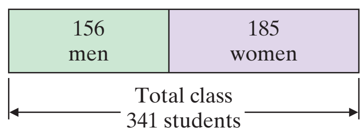
(b) There are 156 men and 185 women taking psychology this semester. Describe the part of the class that consists of women.

(c) Marcel Jones found in the Northwest Territories moose count that five-eighths of the moose observed were female.

Solution

(a) The baseball player got a hit $\frac{5}{12}$ of his times at bat.

(b) The total class is $156 + 185 = 341$. The fractional part that is women is 185 out of 341. Thus $\frac{185}{341}$ of the class is women.



(c) Five-eighths of the moose observed were female. The fraction is $\frac{5}{8}$.

Practice Problem 3 Use a fraction to describe each situation.

(a) 9 out of the 17 players on the basketball team are on the dean's list.

(b) The senior class has 382 men and 351 women. Describe the part of the class consisting of men.

(c) John needed seven-eighths of a metre of material.

EXAMPLE 4 Wanda made 13 calls, out of which she made five sales. Albert made 17 calls, out of which he made six sales. Write a fraction that describes for both people together the number of calls in which a sale was made compared with the total number of calls.

Solution There are $5 + 6 = 11$ calls in which a sale was made.

There were $13 + 17 = 30$ total calls.

Thus $\dfrac{11}{30}$ of the calls resulted in a sale.

Practice Problem 4 An inspector found that one out of seven belts was defective. She also found that two out of nine shirts were defective. Write a fraction that describes what part of all the objects examined were defective.

Developing Your Study Skills

Previewing New Material

Part of your study time each day should consist of looking ahead to those sections in your text that are to be covered the following day. You do not necessarily have to study and learn the material on your own, but if you survey the concepts, terminology, diagrams, and examples, the new ideas will seem more familiar to you when the instructor presents them. You can take note of concepts that appear confusing or difficult and be ready to listen carefully for your instructor's explanations. You can be prepared to ask the questions that will increase your understanding. Previewing new material enables you to see what is coming and prepares you to be ready to absorb it.

Section 1 Exercises

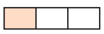
Verbal and Writing Skills

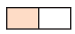
1. A __________ can be used to represent part of a whole or part of a group.
2. In a fraction, the __________ tells the number of parts we are interested in.
3. In a fraction, the __________ tells the total number of parts in the whole or in the group.
4. Describe a real-life situation that involves fractions.

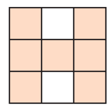
Name the numerator and the denominator in each fraction.

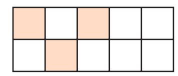
5. $\frac{3}{5}$ **6.** $\frac{9}{11}$ **7.** $\frac{7}{8}$ **8.** $\frac{9}{10}$ **9.** $\frac{1}{17}$ **10.** $\frac{1}{15}$

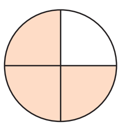
In exercises 11–30, use a fraction to represent the shaded part of the object or the shaded portion of the set of objects.

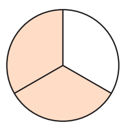
11. 

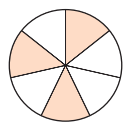
12. 

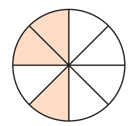
13. 

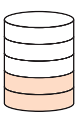
14. 

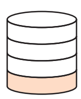
15. 

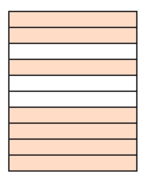
16. 

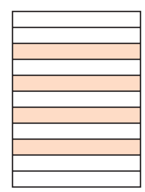
17. 

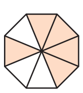
18. 

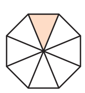
19. 

20. 

21. 

22. 

23. 

24. 

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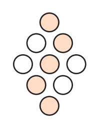
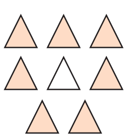
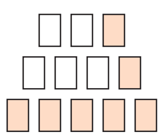
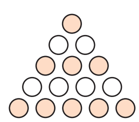
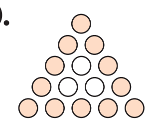
26. 

27. 

28. 

29. 

30. 

Draw a sketch to illustrate each fractional part.

31. $\frac{1}{5}$ of an object **32.** $\frac{3}{7}$ of an object **33.** $\frac{3}{8}$ of an object

34. $\frac{5}{12}$ of an object **35.** $\frac{7}{10}$ of an object **36.** $\frac{5}{9}$ of an object

Applications

37. ***Anthropology Class*** Professor Sousa has 83 students in her anthropology lecture class. Forty-two of the students are sophomores and the others are juniors. What fraction of the class is sophomores?

38. ***Personal Finance*** Miguel bought a notebook with a total purchase price of 98¢. Of this amount, 7¢ was sales tax. What fractional part of the total purchase price was sales tax?

39. ***Personal Finance*** Lance bought a 100-CD jukebox for $750. Part of it was paid for with the $209 he earned parking cars for the valet service at a local wedding reception hall. What fractional part of the jukebox was paid for by his weekend earnings?

40. ***Personal Finance*** Jillian earned $165 over the weekend at her waitressing job. She used $48 of it to repay a loan to her sister. What fractional part of her earnings did Jillian use to repay her sister?

41. ***Political Campaigns*** The Conservative Party fundraising event served 122 chicken dinners and 89 roast beef dinners to its contributors. What fractional part of the guests ate roast beef?

42. ***Education*** George Brown College has 78 full-time instructors and 31 part-time instructors. What fractional part of the faculty are part-time?

43. ***Selling Trees*** Boy Scout Troop #33 had a Christmas tree sale to raise money for a summer camping trip. In one afternoon, they sold 9 balsam firs, 12 Norwegian pines, and 5 Douglas firs. What fractional part of the trees sold were balsam firs?

44. ***Animal Shelters*** At the local animal shelter there are 12 puppies, 25 adult dogs, 14 kittens, and 31 adult cats. What fractional part of the animals are either puppies or adult dogs?

45. ***Book Collection*** Marie has 9 novels, 4 biographies, 12 mysteries, and 15 magazines on her bookshelf. What fractional part of the reading material is either novels or magazines?

46. ***Music Collection*** A box of compact discs contains 5 classical CDs, 6 jazz CDs, 4 soundtracks, and 24 blues CDs. What fractional part of the total CDs is either jazz or blues?

47. ***Manufacturing*** The West Peabody Engine Company manufactured two items last week: 101 engines and 94 lawn mowers. It was discovered that 19 engines and 3 lawn mowers were defective. Of the engines that were not defective, 40 were properly constructed but 42 were not of the highest quality. Of the lawn mowers that were not defective, 50 were properly constructed but 41 were not of the highest quality.

(a) What fractional part of all items manufactured was of the highest quality?

(b) What fractional part of all items manufactured was defective?

48. ***Tour Bus*** A Chicago tour bus held 25 women and 33 men. 12 women wore jeans. 19 men wore jeans. In the group of 25 women, a subgroup of 8 women wore sandals. In the group of 19 men, a subgroup of 10 wore sandals.

(a) What fractional part of the people on the bus wore jeans?

(b) What fractional part of the women on the bus wore sandals?

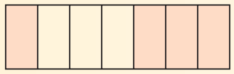
To Think About

49. Illustrate a real-life example of the fraction $\frac{0}{6}$.

50. What happens when we try to illustrate a real-life example of the fraction $\frac{6}{0}$? Why?

Quick Quiz 1

1. Use a fraction to represent the shaded part of the object.



2. Fleming College has 371 students taking classes on Monday night. Of those students, 204 drive a car to campus. Write a fraction that describes the part of the Monday night students who drive a car to class.

3. At the YMCA at 10 P.M. last Friday, 8 men were lifting weights and 5 women were lifting weights. At the same time, 7 men were riding stationary bikes and 13 women were riding stationary bikes. No other people were in the gym at that time. What fractional part of the people in the gym were lifting weights?

4. **Concept Check** One hundred twenty new businesses have opened in Springfield in the last five years. Sixty-five of them were restaurants; the remaining ones were not. Thirty new restaurants went out of business; the other new restaurants did not. Of all the new businesses that were not restaurants, 25 of them went out of business; the others did not. Explain how you can find a fraction that represents the fractional part of the new businesses that did not go out of business.

Section 2 Simplifying Fractions

1 Writing a Number as a Product of Prime Factors

A **prime number** is a whole number greater than 1 that cannot be evenly divided except by itself and 1. If you examine all the whole numbers from 1 to 50, you will find 15 prime numbers.

THE FIRST 15 PRIME NUMBERS

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

A **composite number** is a whole number greater than 1 that can be divided by whole numbers other than 1 and itself. The number 12 is a composite number.

$$12 = 2 \times 6 \qquad \text{and} \qquad 12 = 3 \times 4$$

The number 1 is neither a prime nor a composite number. The number 0 is neither a prime nor a composite number.

Recall that factors are numbers that are multiplied together. Prime factors are prime numbers. To check to see if a number is prime or composite, simply divide the smaller primes (such as 2, 3, 5, 7, 11, ...) into the given number. If the number can be divided exactly without a remainder by one of the smaller primes, it is a composite and not a prime.

Some students find the following rules helpful when deciding if a number can be divided by 2, 3, or 5.

DIVISIBILITY TESTS

1. A number is divisible by 2 if the last digit is 0, 2, 4, 6, or 8.
2. A number is divisible by 3 if the sum of the digits is divisible by 3.
3. A number is divisible by 5 if the last digit is 0 or 5.

To illustrate:

1. 478 is divisible by 2 since it ends in 8.
2. 531 is divisible by 3 since when we add the digits of 531 $(5 + 3 + 1)$ we get 9, which is divisible by 3.
3. 985 is divisible by 5 since it ends in 5.

EXAMPLE 1 Write each whole number as the product of prime factors.

(a) 12 **(b)** 60 **(c)** 168

Solution

(a) To start, write 12 as the product of any two factors. We will write 12 as 4×3.

$12 = 4 \times 3$ Now check whether the factors are prime. If not, factor these.

$2 \times 2 \times 3$

$12 = 2 \times 2 \times 3$ Now all factors are prime, so 12 is completely factored.

Student Learning Objectives

After studying this section, you will be able to:

1. Write a number as a product of prime factors.
2. Reduce a fraction to lowest terms.
3. Determine whether two fractions are equal.

Instead of writing $2 \times 2 \times 3$, we can write $2^2 \times 3$.

Note: To start, we could write 12 as 2×6. Begin this way and follow the preceding steps. Is the product of prime factors the same? Will this always be true?

(b) We follow the same steps as in (a).

$$60 = 6 \times 10$$

$$3 \times 2 \times 2 \times 5$$

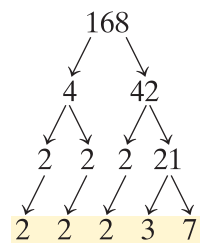
Check that all factors are prime.

$$60 = 2 \times 2 \times 3 \times 5$$

Instead of writing $2 \times 2 \times 3 \times 5$, we can write $2^2 \times 3 \times 5$.

Note that in the final answer the prime factors are listed in order from least to greatest.

(c) Some students like to use a **factor tree** to help write a number as a product of prime factors as illustrated below.



$$168 = 2 \times 2 \times 2 \times 3 \times 7$$

or $$168 = 2^3 \times 3 \times 7$$

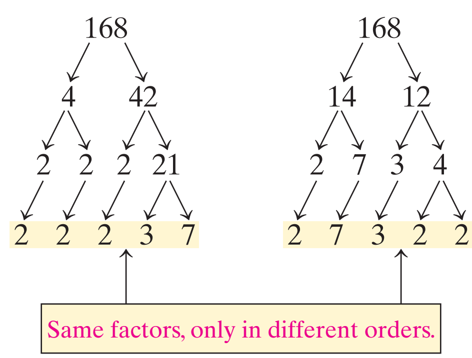
Practice Problem 1 Write each whole number as a product of primes.

(a) 18 **(b)** 72 **(c)** 400

NOTE TO STUDENT: Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.

Suppose we started Example 1(c) by writing $168 = 14 \times 12$. Would we get the same answer? Would our answer be correct? Let's compare.

Again we will use a factor tree.



Thus $168 = 2 \times 2 \times 2 \times 3 \times 7$

or $= 2^3 \times 3 \times 7.$

The order of prime factors is not important because multiplication is commutative. No matter how we start, when we factor a composite number, we always get exactly the same prime factors.

THE FUNDAMENTAL THEOREM OF ARITHMETIC

Every composite number can be written in exactly one way as a product of prime numbers.

We have seen this in our Solution to Example 1(c).

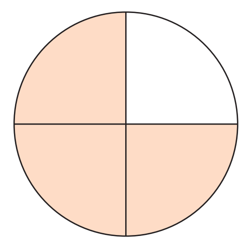
You will be able to check this theorem again in Section 2 Exercises, exercises 7–26. Writing a number as a product of prime factors is also called **prime factorization.**

2 Reducing a Fraction to Lowest Terms

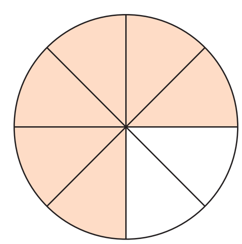
You know that $5 + 2$ and $3 + 4$ are two ways to write the same number. We say they are *equivalent* because they are *equal* to the same *value.* They are both ways of writing the value 7.

Like whole numbers, fractions can be written in more than one way. For example, $\frac{2}{4}$ and $\frac{1}{2}$ are two ways to write the same number. The value of the fractions is the same. When we use fractions, we often need to write them in another form. If we make the numerator and denominator smaller, we *simplify* the fractions.

Compare the two fractions in the drawings on the right. In each picture the shaded part is the same size. The fractions $\frac{3}{4}$ and $\frac{6}{8}$ are called **equivalent fractions.** The fraction $\frac{3}{4}$ is in **simplest form.** To see how we can change $\frac{6}{8}$ to $\frac{3}{4}$, we look at a property of the number 1.



$\frac{3}{4}$ of the circle is shaded.



$\frac{6}{8}$ of the circle is shaded.

Any nonzero number divided by itself is 1.

$$\frac{5}{5} = \frac{17}{17} = \frac{c}{c} = 1$$

Thus, if we multiply a fraction by $\frac{5}{5}$ or $\frac{17}{17}$ or $\frac{c}{c}$ (remember, c cannot be zero), the value of the fraction is unchanged because we are multiplying by a form of 1. We can use this rule to show that $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent.

$$\frac{3}{4} \times \frac{2}{2} = \frac{6}{8}$$

In general, if b and c are not zero,

$$\frac{a}{b} = \frac{a \times c}{b \times c}$$

To reduce a fraction, we find a **common factor** in the numerator and in the denominator and divide it out. In the fraction $\frac{6}{8}$, the common factor is 2.

$$\frac{6}{8} = \frac{3 \times \overset{1}{\cancel{2}}}{4 \times \underset{1}{\cancel{2}}} = \frac{3}{4}$$

$$\frac{6}{8} = \frac{3}{4}$$

For all fractions (where a, b, and c are not zero), if c is a common factor,

$$\frac{a}{b} = \frac{a \div c}{b \div c}$$

A fraction is called **simplified, reduced,** or **in lowest terms** if the numerator and the denominator have only 1 *as a common factor.*

EXAMPLE 2 Simplify (write in lowest terms).

(a) $\frac{15}{25}$ **(b)** $\frac{42}{56}$

Solution

(a) $\frac{15}{25} = \frac{15 \div 5}{25 \div 5} = \frac{3}{5}$ The greatest common factor is 5. Divide the numerator and the denominator by 5.

(b) $\frac{42}{56} = \frac{42 \div 14}{56 \div 14} = \frac{3}{4}$ The greatest common factor is 14. Divide the numerator and the denominator by 14.

Perhaps 14 was not the first common factor you thought of. Perhaps you did see the common factor 2. Divide out 2. Then look for another common factor, 7. Now divide out 7.

$$\frac{42}{56} = \frac{42 \div 2}{56 \div 2} = \frac{21}{28} = \frac{21 \div 7}{21 \div 7} = \frac{3}{4}$$

If we do not see large factors at first, sometimes we can simplify a fraction by dividing both numerator and denominator by a smaller common factor several times, until no common factors are left.

***NOTE TO STUDENT:** Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.*

Practice Problem 2 Simplify by dividing out common factors.

(a) $\frac{30}{42}$ **(b)** $\frac{60}{132}$

A second method to reduce or simplify fractions is called the *method of prime factors*. We factor the numerator and the denominator into prime numbers. We then divide the numerator and the denominator by any common prime factors.

EXAMPLE 3 Simplify the fractions by the method of prime factors.

(a) $\frac{35}{42}$ **(b)** $\frac{22}{110}$

Solution

(a) $\frac{35}{42} = \frac{5 \times 7}{2 \times 3 \times 7}$ We factor 35 and 42 into prime factors. The common prime factor is 7.

$= \frac{5 \times \overset{1}{\cancel{7}}}{2 \times 3 \times \underset{1}{\cancel{7}}}$ Now we divide out 7.

$= \frac{5 \times 1}{2 \times 3 \times 1} = \frac{5}{6}$ We multiply the factors in the numerator and denominator to write the reduced or simplified form.

Thus $\frac{35}{42} = \frac{5}{6}$, and $\frac{5}{6}$ is the simplified form.

(b) $\frac{22}{110} = \frac{2 \times 11}{2 \times 5 \times 11} = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{11}}}{\underset{1}{\cancel{2}} \times 5 \times \underset{1}{\cancel{11}}} = \frac{1}{5}$

Practice Problem 3 Simplify the fractions by the method of prime factors.

(a) $\dfrac{120}{135}$ **(b)** $\dfrac{715}{880}$

NOTE TO STUDENT: Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.

3 Determining Whether Two Fractions Are Equal

After we simplify, how can we check that a reduced fraction is *equivalent* to the original fraction? If two fractions are equal, their diagonal products or **cross products** are equal. This is called the **equality test for fractions.** If $\frac{3}{4} = \frac{6}{8}$, then

$$\frac{3}{4} \stackrel{?}{=} \frac{6}{8} \quad \begin{aligned} &\longrightarrow 4 \times 6 = 24 \\ &\longrightarrow 3 \times 8 = 24 \end{aligned}$$

Products are equal.

If two fractions are unequal (we use the symbol $\neq$), their *cross* products are unequal. If $\dfrac{5}{6} \neq \dfrac{6}{7}$, then

$$\frac{5}{6} \stackrel{?}{=} \frac{6}{7} \quad \begin{aligned} &\longrightarrow 6 \times 6 = 36 \\ &\longrightarrow 5 \times 7 = 35 \end{aligned}$$

Products are not equal.

Since $36 \neq 35$, we know that $\dfrac{5}{6} \neq \dfrac{6}{7}$. The test can be described in this way.

EQUALITY TEST FOR FRACTIONS

For any two fractions where a, b, c, and d are whole numbers and $b \neq 0$, $d \neq 0$, if $\dfrac{a}{b} = \dfrac{c}{d}$, then $a \times d = b \times c$.

EXAMPLE 4 Are these fractions equal? Use the equality test.

(a) $\dfrac{2}{11} \stackrel{?}{=} \dfrac{18}{99}$ **(b)** $\dfrac{3}{16} \stackrel{?}{=} \dfrac{12}{62}$

Solution

(a)

$$\frac{2}{11} \stackrel{?}{=} \frac{18}{99} \quad \begin{aligned} &\longrightarrow 11 \times 18 = 198 \\ &\longrightarrow 2 \times 99 = 198 \end{aligned}$$

Products are equal.

Since $198 = 198$, we know that $\dfrac{2}{11} = \dfrac{18}{99}$.

(b)

$$\frac{3}{16} \stackrel{?}{=} \frac{12}{62} \quad \begin{aligned} &\longrightarrow 16 \times 12 = 192 \\ &\longrightarrow 3 \times 62 = 186 \end{aligned}$$

Products are not equal.

Since $192 \neq 186$, we know that $\dfrac{3}{16} \neq \dfrac{12}{62}$.

Practice Problem 4 Test whether the following fractions are equal.

(a) $\dfrac{84}{108} \stackrel{?}{=} \dfrac{7}{9}$ **(b)** $\dfrac{3}{7} \stackrel{?}{=} \dfrac{79}{182}$

Section 2 Exercises

Verbal and Writing Skills

1. Which of these whole numbers are prime?
4, 12, 11, 15, 6, 19, 1, 41, 38, 24, 5, 46

2. A prime number is a whole number greater than 1 that cannot be evenly ________ except by itself and 1.

3. A ________ ________ is a whole number greater than 1 that can be divided by whole numbers other than itself and 1.

4. Every composite number can be written in exactly one way as a ________ of ________ numbers.

5. Give an example of a composite number written as a product of primes.

6. Give an example of equivalent (equal) fractions.

Write each number as a product of prime factors.

7. 15 **8.** 9 **9.** 35 **10.** 8 **11.** 49

12. 30 **13.** 16 **14.** 81 **15.** 55 **16.** 42

17. 63 **18.** 48 **19.** 84 **20.** 125 **21.** 54

22. 99 **23.** 120 **24.** 135 **25.** 184 **26.** 216

Determine which of these whole numbers are prime. If a number is composite, write it as the product of prime factors.

27. 47 **28.** 31 **29.** 57 **30.** 51

31. 67 **32.** 71 **33.** 62 **34.** 91

35. 89 **36.** 97 **37.** 127 **38.** 119

39. 121 **40.** 95 **41.** 129 **42.** 143

Reduce each fraction by finding a common factor in the numerator and in the denominator and dividing by the common factor.

43. $\frac{18}{27}$ **44.** $\frac{16}{24}$ **45.** $\frac{36}{48}$ **46.** $\frac{28}{49}$

47. $\frac{63}{90}$ **48.** $\frac{45}{75}$ **49.** $\frac{210}{310}$ **50.** $\frac{110}{140}$

Reduce each fraction by the method of prime factors.

51. $\frac{3}{15}$ **52.** $\frac{7}{21}$ **53.** $\frac{66}{88}$ **54.** $\frac{42}{56}$

55. $\frac{30}{45}$ **56.** $\frac{65}{91}$ **57.** $\frac{60}{75}$ **58.** $\frac{42}{70}$

Mixed Practice *Reduce each fraction by any method.*

59. $\frac{33}{36}$ **60.** $\frac{40}{96}$ **61.** $\frac{63}{108}$ **62.** $\frac{72}{132}$ **63.** $\frac{88}{121}$

64. $\frac{125}{200}$ **65.** $\frac{120}{200}$ **66.** $\frac{200}{300}$ **67.** $\frac{220}{260}$ **68.** $\frac{210}{390}$

Are these fractions equal? Why or why not?

69. $\frac{4}{16} \stackrel{?}{=} \frac{7}{28}$ **70.** $\frac{10}{65} \stackrel{?}{=} \frac{2}{13}$ **71.** $\frac{12}{40} \stackrel{?}{=} \frac{3}{13}$ **72.** $\frac{24}{72} \stackrel{?}{=} \frac{15}{45}$

73. $\frac{23}{27} \stackrel{?}{=} \frac{92}{107}$ **74.** $\frac{70}{120} \stackrel{?}{=} \frac{41}{73}$ **75.** $\frac{27}{57} \stackrel{?}{=} \frac{45}{95}$

76. $\frac{18}{24} \stackrel{?}{=} \frac{23}{28}$ **77.** $\frac{60}{95} \stackrel{?}{=} \frac{12}{19}$ **78.** $\frac{21}{27} \stackrel{?}{=} \frac{112}{144}$

Applications *Reduce the fractions in your answers.*

79. ***Pizza Delivery*** Pizza Palace made 128 deliveries on Saturday night. The manager found that 32 of the deliveries were of more than one pizza. He wanted to study the deliveries that consisted of just one pizza. What fractional part of the deliveries were of just one pizza?

80. ***Medical Students*** Medical students frequently work long hours. Susan worked a 16-hour shift, spending 12 hours in the emergency room and 4 hours in surgery. What fractional part of her shift was she in the emergency room? What fractional part of her shift was she in surgery?

81. ***Teaching*** Professor Nguyen found that 12 out of 96 students in his Aspects of Chemistry course failed the first exam. What fractional part of the class failed the exam? What fractional part of the class passed?

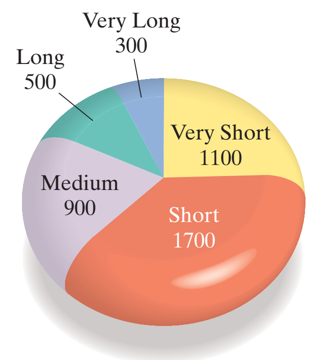
82. ***Wireless Communications*** William works for a wireless communications company that makes beepers and mobile phones. He inspected 315 beepers and found that 20 were defective. What fractional part of the beepers were not defective?

83. ***Personal Finance*** Amelia earned $8400 during her summer vacation. She saved $6000 of her earnings for a trip to New Zealand. What fractional part of her earnings did she save for her trip?

84. ***Real Estate*** Monique's sister and her husband have been working two jobs each to put a down payment on a plot of land where they plan to build their house. The purchase price is $42 500. They have saved $5500. What fractional part of the cost of the land have they saved?

Education *The following data was compiled on the students attending day classes at North Shore Community College.*

Number of Students	Daily Distance Travelled from Home to College (kilometres)	Length of Commute
1100	0–6	Very short
1700	7–12	Short
900	13–18	Medium
500	19–24	Long
300	More than 24	Very long



The number of students with each type of commute is displayed in the circle graph to the right.

Answer exercises 85–88 based on the preceding data. Reduce all fractions in your answers.

85. What fractional part of the student body has a short daily commute to the college?

86. What fractional part of the student body has a medium daily commute to the college?

87. What fractional part of the student body has a long or very long daily commute to the college?

88. What fractional part of the student body has a daily commute to the college that is considered less than long?

Quick Quiz 2 Reduce each fraction.

1. $\frac{25}{35}$

2. $\frac{14}{84}$

3. $\frac{105}{40}$

4. **Concept Check** Explain how you would determine if the fraction $\frac{195}{231}$ can be reduced.

SECTION 3 CONVERTING BETWEEN IMPROPER FRACTIONS AND MIXED NUMBERS

1 Changing a Mixed Number to an Improper Fraction

We have names for different kinds of fractions. If the value of a fraction is less than 1, we say the fraction is proper.

$$\frac{3}{5}, \frac{5}{7}, \frac{1}{8} \quad \text{are called } \textbf{proper fractions}.$$

Notice that the numerator is less than the denominator. If the numerator is less than the denominator, the fraction is a proper fraction.

If the value of a fraction is greater than or equal to 1, the quantity can be written as an improper fraction or as a mixed number.

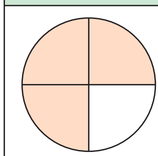
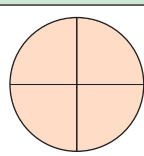
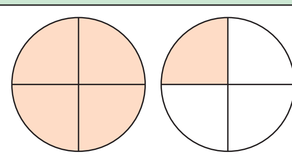
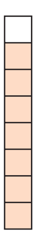
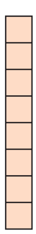
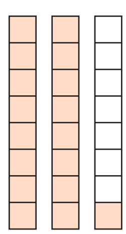
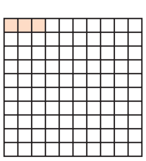
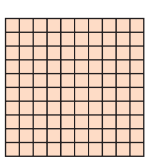
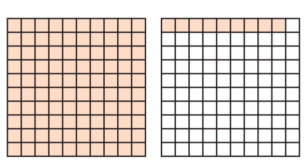
Suppose that we have 1 whole pizza and $\frac{1}{6}$ of a pizza. We could write this as $1\frac{1}{6}$. $1\frac{1}{6}$ is called a mixed number. A **mixed number** is the sum of a whole number greater than zero and a proper fraction. The notation $1\frac{1}{6}$ actually means $1 + \frac{1}{6}$. The plus sign is not usually shown.

Another way of writing $1\frac{1}{6}$ pizza is to write $\frac{7}{6}$ pizza. $\frac{7}{6}$ is called an improper fraction. Notice that the numerator is greater than the denominator. If the numerator is greater than or equal to the denominator, the fraction is an improper fraction.

$$\frac{7}{6}, \frac{6}{6}, \frac{5}{4}, \frac{8}{3}, \frac{2}{2} \quad \text{are } \textbf{improper fractions.}$$

The following chart will help you visualize these different fractions and their names.

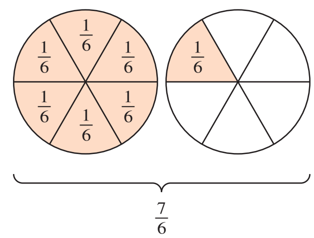
Because in some cases improper fractions are easier to add, subtract, multiply, and divide than mixed numbers, we often change mixed numbers to improper fractions when we perform calculations with them.

Value Less Than 1	Value Equal To 1	Value Greater Than 1
Proper Fraction	Improper Fraction	Improper Fraction or Mixed Number
 $\frac{3}{4}$	 $\frac{4}{4}$	 $\frac{5}{4}$ or $1\frac{1}{4}$
 $\frac{7}{8}$	 $\frac{8}{8}$	 $\frac{17}{8}$ or $2\frac{1}{8}$
 $\frac{3}{100}$	 $\frac{100}{100}$	 $\frac{109}{100}$ or $1\frac{9}{100}$

Student Learning Objectives

After studying this section, you will be able to:

1. Change a mixed number to an improper fraction.
2. Change an improper fraction to a mixed number.
3. Reduce a mixed number or an improper fraction to lowest terms.



CHANGING A MIXED NUMBER TO AN IMPROPER FRACTION

1. Multiply the whole number by the denominator of the fraction.
2. Add the numerator of the fraction to the product found in step 1.
3. Write the sum found in step 2 over the denominator of the fraction.

EXAMPLE 1 Change each mixed number to an improper fraction.

(a) $3\frac{2}{5}$ **(b)** $5\frac{4}{9}$ **(c)** $18\frac{3}{5}$

Solution

Multiply the whole number by the denominator.

Add the numerator to the product.

(a) $3\frac{2}{5} = \frac{3 \times 5 + 2}{5} = \frac{15 + 2}{5} = \frac{17}{5}$

Write the sum over the denominator.

(b) $5\frac{4}{9} = \frac{5 \times 9 + 4}{9} = \frac{45 + 4}{9} = \frac{49}{9}$

(c) $18\frac{3}{5} = \frac{18 \times 5 + 3}{5} = \frac{90 + 3}{5} = \frac{93}{5}$

NOTE TO STUDENT: *Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.*

Practice Problem 1 Change the mixed numbers to improper fractions.

(a) $4\frac{3}{7}$ **(b)** $6\frac{2}{3}$ **(c)** $19\frac{4}{7}$

2 Changing an Improper Fraction to a Mixed Number

We often need to change an improper fraction to a mixed number.

CHANGING AN IMPROPER FRACTION TO A MIXED NUMBER

1. Divide the numerator by the denominator.
2. Write the quotient followed by the fraction with the remainder over the denominator.

$$\text{quotient}\ \frac{\text{remainder}}{\text{denominator}}$$

EXAMPLE 2 Write each improper fraction as a mixed number.

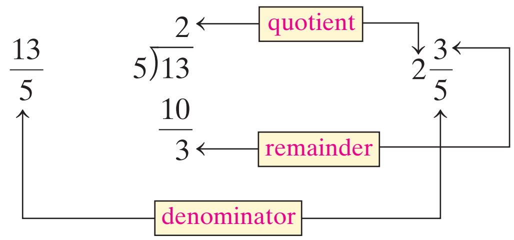
(a) $\frac{13}{5}$ **(b)** $\frac{29}{7}$ **(c)** $\frac{105}{31}$ **(d)** $\frac{85}{17}$

Solution

(a) We divide the denominator 5 into 13.

$$\begin{array}{r} 2 \\ 5\overline{)13} \\ \underline{10} \\ 3 \end{array} \quad \begin{array}{l} \longleftarrow \text{quotient} \\ \\ \\ \longleftarrow \text{remainder} \end{array}$$

The answer is in the form quotient $\dfrac{\text{remainder}}{\text{denominator}}$.



Thus $\dfrac{13}{5} = 2\dfrac{3}{5}$.

(b) $\begin{array}{r} 4 \\ 7\overline{)29} \\ \underline{28} \\ 1 \end{array} \qquad \dfrac{29}{7} = 4\dfrac{1}{7}$

(c) $\begin{array}{r} 3 \\ 31\overline{)105} \\ \underline{93} \\ 12 \end{array} \qquad \dfrac{105}{31} = 3\dfrac{12}{31}$

(d) $\begin{array}{r} 5 \\ 17\overline{)85} \\ \underline{85} \\ 0 \end{array}$ The remainder is 0, so $\dfrac{85}{17} = 5$, a whole number.

Practice Problem 2 Write as a mixed number or a whole number.

(a) $\dfrac{17}{4}$ **(b)** $\dfrac{36}{5}$ **(c)** $\dfrac{116}{27}$ **(d)** $\dfrac{91}{13}$

***NOTE TO STUDENT:* Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.**

3 Reducing a Mixed Number or an Improper Fraction to Lowest Terms

Mixed numbers and improper fractions may need to be reduced if they are not in simplest form. Recall that we write the fraction in terms of prime factors. Then we look for common factors in the numerator and the denominator of the fraction. Then we divide the numerator and the denominator by the common factor.

EXAMPLE 3 Reduce the improper fraction. $\dfrac{22}{8}$

Solution

$$\frac{22}{8} = \frac{\overset{1}{\cancel{2}} \times 11}{\underset{1}{\cancel{2}} \times 2 \times 2} = \frac{11}{4}$$

Practice Problem 3 Reduce the improper fraction.

$$\frac{51}{15}$$

EXAMPLE 4 Reduce the mixed number. $4\frac{21}{28}$

Solution We cannot reduce the whole number 4, only the fraction $\frac{21}{28}$.

$$\frac{21}{28} = \frac{3 \times \cancel{7}^{1}}{4 \times \cancel{7}_{1}} = \frac{3}{4}$$

Therefore, $4\frac{21}{28} = 4\frac{3}{4}$.

Practice Problem 4 Reduce the mixed number.

$$3\frac{16}{80}$$

If an improper fraction contains a very large numerator and denominator, it is best to change the fraction to a mixed number before reducing.

EXAMPLE 5 Reduce $\frac{945}{567}$ by first changing to a mixed number.

Solution

$$\begin{array}{r} 1 \\ 567\overline{)945} \\ \underline{567} \\ 378 \end{array} \qquad \text{so} \quad \frac{945}{567} = 1\frac{378}{567}$$

To reduce the fraction we write

$$\frac{378}{567} = \frac{2 \times 3 \times 3 \times 3 \times 7}{3 \times 3 \times 3 \times 3 \times 7} = \frac{2 \times \cancel{3}^{1} \times \cancel{3}^{1} \times \cancel{3}^{1} \times \cancel{7}^{1}}{3 \times \cancel{3}_{1} \times \cancel{3}_{1} \times \cancel{3}_{1} \times \cancel{7}_{1}} = \frac{2}{3}$$

So $\frac{945}{567} = 1\frac{378}{567} = 1\frac{2}{3}$.

Problems like Example 5 can be done in several different ways. It is not necessary to follow these exact steps when reducing this fraction.

Practice Problem 5 Reduce $\frac{1001}{572}$ by first changing to a mixed number.

TO THINK ABOUT: When a Denominator Is Prime A student concluded that just by looking at the denominator he could tell that the fraction $\frac{1655}{97}$ cannot be reduced unless $1655 \div 97$ is a whole number. How did he come to that conclusion?

Note that 97 is a prime number. The only factors of 97 are 97 and 1. Therefore, *any* fraction with 97 in the denominator can be reduced only if 97 is a factor of the numerator. Since $1655 \div 97$ is not a whole number (see the following division), it is therefore impossible to reduce $\frac{1655}{97}$.

$$\begin{array}{r} 17 \\ 97\overline{)1655} \\ \underline{97} \\ 685 \\ \underline{679} \\ 6 \end{array}$$

You may explore this idea in Section 3 Exercises, exercises 83 and 84.

Section 3 Exercises

Verbal and Writing Skills

1. Describe in your own words how to change a mixed number to an improper fraction.

2. Describe in your own words how to change an improper fraction to a mixed number.

Change each mixed number to an improper fraction.

3. $2\frac{1}{3}$ **4.** $2\frac{3}{4}$ **5.** $2\frac{3}{7}$ **6.** $3\frac{3}{8}$ **7.** $9\frac{2}{9}$ **8.** $8\frac{3}{8}$

9. $10\frac{2}{3}$ **10.** $15\frac{3}{4}$ **11.** $11\frac{3}{5}$ **12.** $15\frac{4}{5}$ **13.** $9\frac{1}{6}$ **14.** $41\frac{1}{2}$

15. $20\frac{1}{6}$ **16.** $6\frac{6}{7}$ **17.** $10\frac{11}{12}$ **18.** $13\frac{5}{7}$ **19.** $7\frac{9}{10}$ **20.** $4\frac{1}{50}$

21. $8\frac{1}{25}$ **22.** $12\frac{5}{6}$ **23.** $5\frac{5}{12}$ **24.** $207\frac{2}{3}$ **25.** $164\frac{2}{3}$ **26.** $33\frac{1}{3}$

27. $8\frac{11}{15}$ **28.** $5\frac{19}{20}$ **29.** $4\frac{13}{25}$ **30.** $5\frac{17}{20}$

Change each improper fraction to a mixed number or a whole number.

31. $\frac{4}{3}$ **32.** $\frac{13}{4}$ **33.** $\frac{11}{4}$ **34.** $\frac{9}{5}$ **35.** $\frac{15}{6}$ **36.** $\frac{23}{6}$

37. $\frac{27}{8}$ **38.** $\frac{80}{5}$ **39.** $\frac{100}{4}$ **40.** $\frac{42}{13}$ **41.** $\frac{86}{9}$ **42.** $\frac{47}{2}$

43. $\frac{70}{3}$ **44.** $\frac{54}{17}$ **45.** $\frac{25}{4}$ **46.** $\frac{19}{3}$ **47.** $\frac{57}{10}$ **48.** $\frac{83}{10}$

49. $\frac{35}{2}$ **50.** $\frac{132}{11}$ **51.** $\frac{91}{7}$ **52.** $\frac{183}{7}$ **53.** $\frac{210}{15}$ **54.** $\frac{196}{9}$

55. $\frac{102}{17}$ **56.** $\frac{104}{8}$ **57.** $\frac{175}{32}$ **58.** $\frac{154}{25}$

Reduce each mixed number.

59. $5\frac{3}{6}$ **60.** $4\frac{6}{8}$ **61.** $4\frac{11}{66}$ **62.** $3\frac{15}{90}$ **63.** $15\frac{18}{72}$ **64.** $10\frac{15}{75}$

Reduce each improper fraction.

65. $\frac{24}{6}$ **66.** $\frac{36}{4}$ **67.** $\frac{36}{15}$ **68.** $\frac{63}{45}$ **69.** $\frac{105}{28}$ **70.** $\frac{112}{21}$

Change to a mixed number and reduce.

71. $\frac{340}{126}$ **72.** $\frac{390}{360}$ **73.** $\frac{580}{280}$

74. $\frac{764}{328}$ **75.** $\frac{508}{296}$ **76.** $\frac{2150}{1000}$

Applications

77. ***Banner Display*** The Science Museum is hanging banners all over the building to commemorate the Apollo astronauts. The art department is using $360\frac{2}{3}$ metres of starry-sky parachute fabric. Change this number to an improper fraction.

78. ***Sculpture*** For the Northwestern University alumni homecoming, the students studying sculpture have made a giant replica of the school using $244\frac{3}{4}$ kilograms of clay. Change this number to an improper fraction.

79. ***Environmental Studies*** A Cape Cod cranberry bog was contaminated by waste from abandoned oil storage tanks at Otis Air Force Base. Damage was done to $\frac{151}{3}$ acres of land. Write this as a mixed number.

80. ***Theatre*** Waite Auditorium needs new velvet stage curtains. The manufacturer took measurements and calculated he would need $\frac{331}{4}$ square metres of fabric. Write this as a mixed number.

81. ***Cooking*** The cafeteria workers at Ipswich High School used $\frac{1131}{8}$ kilograms of flour while cooking for the students last week. Write this as a mixed number.

82. ***Shelf Construction*** The new Centre for Construction Trades and Building Sciences at Algonquin College had several new offices for the faculty and staff. Shelving was constructed for these offices. A total of $\frac{1373}{8}$ metres of shelving was used in the construction. Write this as a mixed number.

To Think About

83. Can $\frac{5687}{101}$ be reduced? Why or why not?

84. Can $\frac{9810}{157}$ be reduced? Why or why not?

Quick Quiz 3

1. Change to an improper fraction.

$$4\frac{7}{13}$$

2. Change to a mixed number.

$$\frac{89}{12}$$

3. Reduce the improper fraction.

$$\frac{42}{14}$$

4. **Concept Check** Explain how you change the mixed number $5\frac{6}{13}$ to an improper fraction.

Section 4 Multiplying Fractions and Mixed Numbers

1 Multiplying Two Fractions That Are Proper or Improper

FUDGE SQUARES

Ingredients:

2 cups sugar	1/4 teaspoon salt
4 oz chocolate	1 teaspoon vanilla
1/2 cup butter	1 cup all-purpose flour
4 eggs	1 cup nutmeats

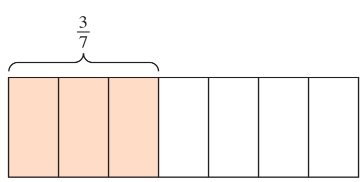
Suppose you want to make an amount equal to half of what the recipe shown will produce. You would multiply the measure given for each ingredient by $\frac{1}{2}$.

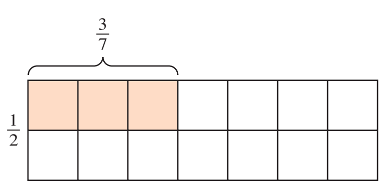
$\frac{1}{2}$ of 2 cups sugar	$\frac{1}{2}$ of $\frac{1}{4}$ teaspoon salt
$\frac{1}{2}$ of 4 oz chocolate	$\frac{1}{2}$ of 1 teaspoon vanilla
$\frac{1}{2}$ of $\frac{1}{2}$ cup butter	$\frac{1}{2}$ of 1 cup all-purpose flour
$\frac{1}{2}$ of 4 eggs	$\frac{1}{2}$ of 1 cup nutmeats

We often use multiplication of fractions to describe taking a fractional part of something. To find $\frac{1}{2}$ of $\frac{3}{7}$, we multiply

$$\frac{1}{2} \times \frac{3}{7} = \frac{3}{14}.$$

We begin with a bar that is $\frac{3}{7}$ shaded. To find $\frac{1}{2}$ of $\frac{3}{7}$ we divide the bar in half and take $\frac{1}{2}$ of the shaded section. $\frac{1}{2}$ of $\frac{3}{7}$ yields 3 out of 14 squares.





Thus $\quad \frac{1}{2} \times \frac{3}{7} = \frac{3}{14}$

When you multiply two proper fractions together, you get a smaller fraction.

To multiply two fractions, we multiply the numerators and multiply the denominators.

$$\frac{2}{3} \times \frac{5}{7} = \frac{10}{21} \begin{array}{l} \leftarrow 2 \times 5 = 10 \\ \leftarrow 3 \times 7 = 21 \end{array}$$

Student Learning Objectives

After studying this section, you will be able to:

1. **Multiply two fractions that are proper or improper.**
2. **Multiply a whole number by a fraction.**
3. **Multiply mixed numbers.**

John Paul Endress/Corbis/Stock Market

MULTIPLICATION OF FRACTIONS

In general, for all positive whole numbers a, b, c, and d,

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}.$$

EXAMPLE 1 Multiply.

(a) $\frac{3}{8} \times \frac{5}{7}$ **(b)** $\frac{1}{11} \times \frac{2}{13}$

Solution

(a) $\frac{3}{8} \times \frac{5}{7} = \frac{3 \times 5}{8 \times 7} = \frac{15}{56}$ **(b)** $\frac{1}{11} \times \frac{2}{13} = \frac{1 \times 2}{11 \times 13} = \frac{2}{143}$

NOTE TO STUDENT: Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.

Practice Problem 1 Multiply.

(a) $\frac{6}{7} \times \frac{3}{13}$ **(b)** $\frac{1}{5} \times \frac{11}{12}$

Some products may be reduced. $\frac{12}{35} \times \frac{25}{18} = \frac{300}{630} = \frac{10}{21}$

By simplifying before multiplication, the reducing can be done more easily. For a multiplication problem, a factor in the numerator can be paired with a common factor in the denominator of the same or a different fraction. We can begin by finding the prime factors in the numerators and denominators. We then divide numerator and denominator by their common prime factors.

EXAMPLE 2 Simplify first and then multiply. $\frac{12}{35} \times \frac{25}{18}$

Solution

$\frac{12}{35} \times \frac{25}{18} = \frac{2 \cdot 2 \cdot 3}{5 \cdot 7} \times \frac{5 \cdot 5}{2 \cdot 3 \cdot 3}$ First we find the prime factors.

$= \frac{2 \cdot 2 \cdot 3 \cdot 5 \cdot 5}{5 \cdot 7 \cdot 2 \cdot 3 \cdot 3}$ Write the product as one fraction.

$= \frac{\overset{1}{\cancel{2}} \cdot 2 \cdot \overset{1}{\cancel{3}} \cdot \overset{1}{\cancel{5}} \cdot 5}{\underset{1}{\cancel{2}} \cdot \underset{1}{\cancel{3}} \cdot 3 \cdot \underset{1}{\cancel{5}} \cdot 7}$ Arrange the factors in order and divide the numerator and denominator by the common factors.

$= \frac{10}{21}$ Multiply the remaining factors.

Practice Problem 2 Simplify first and then multiply.

$$\frac{55}{72} \times \frac{16}{33}$$

Note: Although finding the prime factors of the numerators and denominators will help you avoid errors, you can also begin these problems by dividing the numerators and denominators by larger common factors. This method will be used for the remainder of the exercises in this section.

2 Multiplying a Whole Number by a Fraction

When multiplying a fraction by a whole number, it is more convenient to express the whole number as a fraction with a denominator of 1. We know that $5 = \frac{5}{1}$, $7 = \frac{7}{1}$, and so on.

EXAMPLE 3 Multiply.

(a) $5 \times \dfrac{3}{8}$ **(b)** $\dfrac{22}{7} \times 14$

Solution

(a) $5 \times \dfrac{3}{8} = \dfrac{5}{1} \times \dfrac{3}{8} = \dfrac{15}{8}$ or $1\dfrac{7}{8}$ **(b)** $\dfrac{22}{7} \times 14 = \dfrac{22}{\underset{1}{\cancel{7}}} \times \dfrac{\overset{2}{\cancel{14}}}{1} = \dfrac{44}{1} = 44$

Practice Problem 3 Multiply.

(a) $7 \times \dfrac{5}{13}$ **(b)** $\dfrac{13}{4} \times 8$

EXAMPLE 4 Mr. and Mrs. Jones found that $\frac{2}{7}$ of their income went to pay federal income taxes. Last year they earned \$37 100. How much did they pay in taxes?

Solution We need to find $\frac{2}{7}$ of \$37 100. So we must multiply $\frac{2}{7} \times 37\,100$.

$$\frac{2}{\underset{1}{\cancel{7}}} \times \overset{5300}{\cancel{37\,100}} = \frac{2}{1} \times 5300 = 10\,600$$

They paid \$10 600 in federal income taxes.

Practice Problem 4 Fred and Linda own 98 400 square metres of land. They found that $\frac{3}{8}$ of the land is in a wetland area and cannot be used for building. How many square metres of land are in the wetland area?

3 Multiplying Mixed Numbers

To multiply a fraction by a mixed number or to multiply two mixed numbers, first change each mixed number to an improper fraction.

EXAMPLE 5 Multiply.

(a) $\dfrac{5}{7} \times 3\dfrac{1}{4}$ **(b)** $20\dfrac{2}{5} \times 6\dfrac{2}{3}$ **(c)** $\dfrac{3}{4} \times 1\dfrac{1}{2} \times \dfrac{4}{7}$ **(d)** $4\dfrac{1}{3} \times 2\dfrac{1}{4}$

Solution

(a) $\dfrac{5}{7} \times 3\dfrac{1}{4} = \dfrac{5}{7} \times \dfrac{13}{4} = \dfrac{65}{28}$ or $2\dfrac{9}{28}$

(b) $20\dfrac{2}{5} \times 6\dfrac{2}{3} = \dfrac{\overset{34}{\cancel{102}}}{\underset{1}{\cancel{5}}} \times \dfrac{\overset{4}{\cancel{20}}}{\underset{1}{\cancel{3}}} = \dfrac{136}{1} = 136$

(c) $\frac{3}{4} \times 1\frac{1}{2} \times \frac{4}{7} = \frac{3}{\underset{1}{\cancel{4}}} \times \frac{3}{2} \times \frac{\overset{1}{\cancel{4}}}{7} = \frac{9}{14}$

(d) $4\frac{1}{3} \times 2\frac{1}{4} = \frac{13}{\underset{1}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{4} = \frac{39}{4}$ or $9\frac{3}{4}$

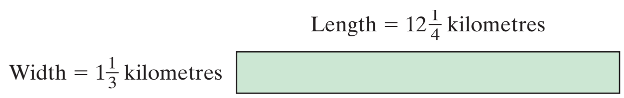
NOTE TO STUDENT: Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.

Practice Problem 5 Multiply.

(a) $2\frac{1}{6} \times \frac{4}{7}$ **(b)** $10\frac{2}{3} \times 13\frac{1}{2}$

(c) $\frac{3}{5} \times 1\frac{1}{3} \times \frac{5}{8}$ **(d)** $3\frac{1}{5} \times 2\frac{1}{2}$

▲ **EXAMPLE 6** Find the area in square kilometres of a rectangle with width $1\frac{1}{3}$ kilometres and length $12\frac{1}{4}$ kilometres.



Solution We find the area of a rectangle by multiplying the width times the length.

$$1\frac{1}{3} \times 12\frac{1}{4} = \frac{\overset{1}{\cancel{4}}}{3} \times \frac{49}{\underset{1}{\cancel{4}}} = \frac{49}{3} \quad \text{or} \quad 16\frac{1}{3}$$

The area is $16\frac{1}{3}$ square kilometres.

▲ **Practice Problem 6** Find the area in square metres of a rectangle with width $1\frac{1}{5}$ metres and length $4\frac{5}{6}$ metres.

EXAMPLE 7 Find the value of x if

$$\frac{3}{7} \cdot x = \frac{15}{42}.$$

Solution The variable x represents a fraction. We know that 3 times one number equals 15 and 7 times another equals 42.

$$\begin{array}{ll} \text{Since } 3 \cdot 5 = 15 & \\ \text{and } \;\; 7 \cdot 6 = 42 & \end{array} \quad \text{we know that} \quad \frac{3}{7} \cdot \frac{5}{6} = \frac{15}{42}.$$

Therefore, $x = \frac{5}{6}$.

Practice Problem 7 Find the value of x if $\frac{8}{9} \cdot x = \frac{80}{81}$.

Section 4 Exercises

Multiply. Make sure all fractions are simplified in the final answer.

1. $\frac{3}{5} \times \frac{7}{11}$

2. $\frac{1}{8} \times \frac{5}{11}$

3. $\frac{3}{4} \times \frac{5}{13}$

4. $\frac{4}{7} \times \frac{3}{5}$

5. $\frac{6}{5} \times \frac{10}{12}$

6. $\frac{7}{8} \times \frac{16}{21}$

7. $\frac{5}{36} \times \frac{9}{20}$

8. $\frac{22}{45} \times \frac{5}{11}$

9. $\frac{12}{25} \times \frac{5}{11}$

10. $\frac{9}{4} \times \frac{13}{27}$

11. $\frac{9}{10} \times \frac{35}{12}$

12. $\frac{12}{17} \times \frac{3}{24}$

13. $8 \times \frac{3}{7}$

14. $\frac{8}{9} \times 6$

15. $\frac{5}{12} \times 8$

16. $5 \times \frac{7}{25}$

17. $\frac{4}{9} \times \frac{3}{7} \times \frac{7}{8}$

18. $\frac{8}{7} \times \frac{5}{12} \times \frac{3}{10}$

19. $\frac{5}{4} \times \frac{9}{10} \times \frac{8}{3}$

20. $\frac{5}{7} \times \frac{15}{2} \times \frac{28}{15}$

Multiply. Change any mixed number to an improper fraction before multiplying.

21. $2\frac{5}{6} \times \frac{3}{17}$

22. $\frac{5}{6} \times 3\frac{3}{5}$

23. $10 \times 3\frac{1}{10}$

24. $12 \times 5\frac{7}{12}$

25. $1\frac{3}{16} \times 0$

26. $0 \times 6\frac{2}{3}$

27. $3\frac{7}{8} \times 1$

28. $\frac{5}{5} \times 11\frac{5}{7}$

29. $1\frac{1}{4} \times 3\frac{2}{3}$

30. $2\frac{3}{5} \times 1\frac{4}{7}$

31. $2\frac{3}{10} \times \frac{3}{5}$

32. $4\frac{3}{5} \times \frac{1}{10}$

33. $4\frac{1}{5} \times 8\frac{1}{3}$

34. $5\frac{1}{4} \times 4\frac{4}{7}$

35. $6\frac{2}{5} \times \frac{1}{4}$

36. $\frac{8}{9} \times 4\frac{1}{11}$

Mixed Practice

Multiply. Make sure all fractions are simplified in the final answer.

37. $\frac{11}{15} \times \frac{35}{33}$

38. $\frac{14}{17} \times \frac{34}{42}$

39. $2\frac{3}{8} \times 5\frac{1}{3}$

40. $4\frac{3}{5} \times 3\frac{3}{4}$

Solve for x.

41. $\frac{4}{9} \cdot x = \frac{28}{81}$

42. $\frac{12}{17} \cdot x = \frac{144}{85}$

43. $\frac{7}{13} \cdot x = \frac{56}{117}$

44. $x \cdot \frac{11}{15} = \frac{77}{225}$

Applications

▲ **45.** ***Geometry*** A spy is running from his captors in a forest that is $8\frac{3}{4}$ kilometres long and $4\frac{1}{3}$ kilometres wide. Find the area of the forest where he is hiding. (*Hint:* The area of a rectangle is the product of the length times the width.)

▲ **46.** ***Geometry*** An area in the Midwest is a designated tornado danger zone. The land is $22\frac{5}{8}$ kilometres long and $16\frac{1}{2}$ kilometres wide. Find the area of the tornado danger zone. (*Hint:* The area of a rectangle is the product of the length times the width.)

47. ***Airplane Travel*** A Learjet airplane has 1360 litres of fuel. The plane averages $1\frac{3}{4}$ kilometres per litre. How far can the plane go?

48. ***Real Estate*** Mel and Sally Hauser bought their house in 1977 for a price of \$56 800. Thirty years later, in 2007, their house was worth $6\frac{1}{2}$ times what they paid for it. How much was Mel and Sally's house worth in 2007?

49. ***Cooking*** A recipe from Nanette's French cookbook for a scalloped potato tart requires $90\frac{1}{2}$ grams of grated cheese. How many grams of cheese would she need if she made one tart for each of her 18 cousins?

▲ **50.** ***Geometry*** The dormitory rooms in Selkirk Hall are being carpeted. Each room requires $20\frac{1}{2}$ square metres of carpet. If there are 30 rooms, how much carpet is needed?

51. ***College Students*** Of the 7998 students at Normandale Community College, $\frac{2}{3}$ of them are under 25 years of age. How many students are under 25 years of age?

52. ***Health Care*** A nurse finds that of the 225 rooms at the Children's Hospital of Eastern Ontario, $\frac{1}{15}$ of them are occupied by surgery patients. How many rooms contain surgery patients?

53. ***Job Search*** Carlos has sent his resumé to 12 064 companies through an Internet job search service. If $\frac{1}{32}$ of the companies e-mail him with an invitation for an interview, how many companies will he have heard from?

54. ***Car Purchase*** Russ purchased a new Buick LeSabre for \$26 500. After one year the car was worth $\frac{4}{5}$ of the purchase price. What was the car worth after one year?

55. ***Jogging*** Mary jogged $4\frac{1}{4}$ kilometres per hour for $1\frac{1}{3}$ hours. During $\frac{1}{3}$ of her jogging time, she was jogging in the rain. How many kilometres did she jog in the rain?

56. ***College Students*** There were 1340 students at the Whitby campus of Durham College during the spring 2009 semester. The registrar discovered that $\frac{2}{5}$ of these students live in the city of Whitby. He further discovered that $\frac{1}{4}$ of the students living in Whitby attend classes only on Monday, Wednesday, and Friday. How many students at the Whitby campus live in the city of Whitby and attend classes only on Monday, Wednesday, and Friday?

To Think About

57. When we multiply two fractions, we look for opportunities to divide a numerator and a denominator by the same number. Why do we bother with that step? Why don't we just multiply the two numerators and the two denominators?

58. Suppose there is an unknown fraction that has *not* been simplified (it is not reduced). You multiply this unknown fraction by $\frac{2}{5}$ and you obtain a simplified answer of $\frac{6}{35}$. How many possible values could this unknown fraction be? Give at least three possible answers.

Quick Quiz 4 Multiply.

1. $32 \times \dfrac{5}{16}$

2. $\dfrac{11}{13} \times \dfrac{4}{5}$

3. $4\dfrac{1}{3} \times 2\dfrac{3}{4}$

4. **Concept Check** Explain how you would multiply the whole number 6 times the mixed number $4\frac{3}{5}$.

SECTION 5 DIVIDING FRACTIONS AND MIXED NUMBERS

Student Learning Objectives

After studying this section, you will be able to:

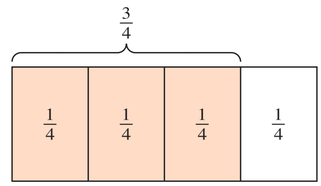
1. **Divide two proper or improper fractions.**
2. **Divide a whole number and a fraction.**
3. **Divide mixed numbers.**

1 Dividing Two Proper or Improper Fractions

Why would you divide fractions? Consider this problem.

- A copper pipe that is $\frac{3}{4}$ of a metre long is to be cut into $\frac{1}{4}$-metre pieces. How many pieces will there be?

To find how many $\frac{1}{4}$'s are in $\frac{3}{4}$, we divide $\frac{3}{4} \div \frac{1}{4}$. We draw a sketch.



Notice that there are three $\frac{1}{4}$'s in $\frac{3}{4}$.

How do we divide two fractions? We **invert** the second fraction and multiply.

$$\frac{3}{4} \div \frac{1}{4} = \frac{3}{\cancel{4}_1} \times \frac{\cancel{4}^1}{1} = \frac{3}{1} = 3$$

When we invert a fraction, we interchange the numerator and the denominator. If we invert $\frac{5}{9}$, we obtain $\frac{9}{5}$. If we invert $\frac{6}{1}$, we obtain $\frac{1}{6}$. Numbers such as $\frac{5}{9}$ and $\frac{9}{5}$ are called **reciprocals** of each other.

RULE FOR DIVISION OF FRACTIONS

To divide two fractions, we invert the second fraction and multiply.

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

(when b, c, and d are not zero).

EXAMPLE 1 Divide.

(a) $\frac{3}{11} \div \frac{2}{5}$ **(b)** $\frac{5}{8} \div \frac{25}{16}$

Solution

(a) $\frac{3}{11} \div \frac{2}{5} = \frac{3}{11} \times \frac{5}{2} = \frac{15}{22}$

(b) $\frac{5}{8} \div \frac{25}{16} = \frac{\cancel{5}^1}{\cancel{8}_1} \times \frac{\cancel{16}^2}{\cancel{25}_5} = \frac{2}{5}$

Practice Problem 1 Divide.

(a) $\frac{7}{13} \div \frac{3}{4}$ **(b)** $\frac{16}{35} \div \frac{24}{25}$

NOTE TO STUDENT: Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.

2 Dividing a Whole Number and a Fraction

When dividing with whole numbers, it is helpful to remember that for any whole number a, $a = \frac{a}{1}$.

EXAMPLE 2 Divide.

(a) $\frac{3}{7} \div 2$ **(b)** $5 \div \frac{10}{13}$

Solution

(a) $\frac{3}{7} \div 2 = \frac{3}{7} \div \frac{2}{1} = \frac{3}{7} \times \frac{1}{2} = \frac{3}{14}$

(b) $5 \div \frac{10}{13} = \frac{5}{1} \div \frac{10}{13} = \frac{\overset{1}{\cancel{5}}}{1} \times \frac{13}{\underset{2}{\cancel{10}}} = \frac{13}{2}$ or $6\frac{1}{2}$

Practice Problem 2 Divide.

(a) $\frac{3}{17} \div 6$ **(b)** $14 \div \frac{7}{15}$

EXAMPLE 3 Divide, if possible.

(a) $\frac{23}{25} \div 1$ **(b)** $1 \div \frac{7}{5}$ **(c)** $0 \div \frac{4}{9}$ **(d)** $\frac{3}{17} \div 0$

Solution

(a) $\frac{23}{25} \div 1 = \frac{23}{25} \times \frac{1}{1} = \frac{23}{25}$

(b) $1 \div \frac{7}{5} = \frac{1}{1} \times \frac{5}{7} = \frac{5}{7}$

(c) $0 \div \frac{4}{9} = \frac{0}{1} \times \frac{9}{4} = \frac{0}{4} = 0$ Zero divided by any nonzero number is zero.

(d) $\frac{3}{17} \div 0$ Division by zero is undefined.

Practice Problem 3 Divide, if possible.

(a) $1 \div \frac{11}{13}$ **(b)** $\frac{14}{17} \div 1$

(c) $\frac{3}{11} \div 0$ **(d)** $0 \div \frac{9}{16}$

SIDELIGHT: Invert and Multiply

Why do we divide by inverting the second fraction and multiplying? What is really going on when we do this? We are actually multiplying by 1. To see why, consider the following.

$$\frac{3}{7} \div \frac{2}{3} = \frac{\frac{3}{7}}{\frac{2}{3}}$$

We write the division by using another fraction bar.

$$= \frac{\frac{3}{7}}{\frac{2}{3}} \times 1$$

Any fraction can be multiplied by 1 without changing the value of the fraction. This is the fundamental rule of fractions.

$$= \frac{\frac{3}{7}}{\frac{2}{3}} \times \frac{\frac{3}{2}}{\frac{3}{2}}$$

Any nonzero number divided by itself equals 1.

$$= \frac{\frac{3}{7} \times \frac{3}{2}}{\frac{2}{3} \times \frac{3}{2}}$$

Definition of multiplication of fractions.

$$= \frac{\frac{3}{7} \times \frac{3}{2}}{1} = \frac{3}{7} \times \frac{3}{2}$$

Any number can be written as a fraction with a denominator of 1 without changing its value.

Thus

$$\frac{3}{7} \div \frac{2}{3} = \frac{3}{7} \times \frac{3}{2} = \frac{9}{14}.$$

3 Dividing Mixed Numbers

If one or more mixed numbers are involved in the division, they should be converted to improper fractions first.

EXAMPLE 4 Divide.

(a) $3\frac{7}{15} \div 1\frac{1}{25}$ **(b)** $\frac{3}{5} \div 2\frac{1}{7}$

Solution

(a) $3\frac{7}{15} \div 1\frac{1}{25} = \frac{52}{15} \div \frac{26}{25} = \frac{\overset{2}{\cancel{52}}}{\underset{3}{\cancel{15}}} \times \frac{\overset{5}{\cancel{25}}}{\underset{1}{\cancel{26}}} = \frac{10}{3}$ or $3\frac{1}{3}$

(b) $\frac{3}{5} \div 2\frac{1}{7} = \frac{3}{5} \div \frac{15}{7} = \frac{\overset{1}{\cancel{3}}}{5} \times \frac{7}{\underset{5}{\cancel{15}}} = \frac{7}{25}$

Practice Problem 4 Divide.

(a) $1\frac{1}{5} \div \frac{7}{10}$ **(b)** $2\frac{1}{4} \div 1\frac{7}{8}$

NOTE TO STUDENT: Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.

The division of two fractions may be indicated by a wide fraction bar.

EXAMPLE 5 Divide.

(a) $\dfrac{10\frac{2}{9}}{2\frac{1}{3}}$ **(b)** $\dfrac{1\frac{1}{15}}{3\frac{1}{3}}$

Solution

(a) $\dfrac{10\frac{2}{9}}{2\frac{1}{3}} = 10\dfrac{2}{9} \div 2\dfrac{1}{3} = \dfrac{92}{9} \div \dfrac{7}{3} = \dfrac{92}{\cancel{9}_{3}} \times \dfrac{\cancel{3}^{1}}{7} = \dfrac{92}{21}$ or $4\dfrac{8}{21}$

(b) $\dfrac{1\frac{1}{15}}{3\frac{1}{3}} = 1\dfrac{1}{15} \div 3\dfrac{1}{3} = \dfrac{16}{15} \div \dfrac{10}{3} = \dfrac{\cancel{16}^{8}}{\cancel{15}_{5}} \times \dfrac{\cancel{3}^{1}}{\cancel{10}_{5}} = \dfrac{8}{25}$

Practice Problem 5 Divide.

(a) $\dfrac{5\frac{2}{3}}{7}$ **(b)** $\dfrac{1\frac{2}{5}}{2\frac{1}{3}}$

Some students may find Example 6 difficult at first. Read it slowly and carefully. It may be necessary to read it several times before it becomes clear.

EXAMPLE 6 Find the value of x if $x \div \frac{8}{7} = \frac{21}{40}$.

Solution First we will change the division problem to an equivalent multiplication problem.

$$x \div \frac{8}{7} = \frac{21}{40}$$

$$x \cdot \frac{7}{8} = \frac{21}{40}$$

x represents a fraction.

In the numerator, we want to know what times 7 equals 21. In the denominator, we want to know what times 8 equals 40.

$$\frac{3}{5} \cdot \frac{7}{8} = \frac{21}{40}$$

Thus $x = \frac{3}{5}$.

Practice Problem 6 Find the value of x if $x \div \frac{3}{2} = \frac{22}{36}$.

EXAMPLE 7 There are 117 milligrams of cholesterol in $4\frac{1}{3}$ cups of milk. How much cholesterol is in 1 cup of milk?

Solution We want to divide the 117 by $4\frac{1}{3}$ to find out how much is in 1 cup.

$$117 \div 4\frac{1}{3} = 117 \div \frac{13}{3} = \frac{\overset{9}{\cancel{117}}}{1} \times \frac{3}{\underset{1}{\cancel{13}}} = \frac{27}{1} = 27$$

Thus there are 27 milligrams of cholesterol in 1 cup of milk.

***NOTE TO STUDENT:* Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.**

Practice Problem 7 A copper pipe that is $19\frac{1}{4}$ metres long will be cut into 14 equal pieces. How long will each piece be?

Take a little time to review Examples 1–7 and Practice Problems 1–7. This is important material. It is crucial to understand how to do each of these problems. Some extra time spent reviewing here will make the homework exercises go much more quickly.

Developing Your Study Skills

Why Is Review Necessary?

You master a course in mathematics by learning the concepts one step at a time. There are basic concepts like addition, subtraction, multiplication, and division of whole numbers that are considered the foundation upon which all of mathematics is built. These must be mastered first. Then the study of mathematics is built step by step upon this foundation, each step supporting the next. The process is a carefully designed procedure, so no steps can be skipped. A student of mathematics needs to realize the importance of this building process to succeed.

Because learning new concepts depends on those previously learned, students often need to take time to review. The reviewing process will strengthen the understanding and application of concepts that are weak due to lack of mastery or passage of time. Review at the right time on the right concepts can strengthen previously learned skills and make progress possible.

Timely, periodic review of previously learned mathematical concepts is absolutely necessary in order to master new concepts. You may have forgotten a concept or grown a bit rusty in applying it. Reviewing is the answer. Make use of any review sections, whether they are assigned or not. Look back to previous material whenever you have forgotten how to do something. Study the examples and practise some exercises to refresh your understanding.

Be sure that you understand and can perform the computations of each new concept. This will enable you to move successfully on to the next ones.

Section 5 Exercises

Make sure all fractions are simplified in the final answer.

Verbal and Writing Skills

1. In your own words explain how to remember that when you divide two fractions you invert the *second* fraction and multiply by the first. How can you be sure that you don't invert the *first* fraction by mistake?

2. Explain why $2 \div \frac{1}{3}$ is a larger number than $2 \div \frac{1}{2}$.

Divide, if possible.

3. $\frac{7}{16} \div \frac{3}{4}$

4. $\frac{3}{13} \div \frac{9}{26}$

5. $\frac{2}{3} \div \frac{4}{27}$

6. $\frac{25}{49} \div \frac{5}{7}$

7. $\frac{7}{18} \div \frac{21}{6}$

8. $\frac{8}{15} \div \frac{24}{35}$

9. $\frac{5}{9} \div \frac{1}{5}$

10. $\frac{3}{4} \div \frac{2}{3}$

11. $\frac{4}{15} \div \frac{4}{15}$

12. $\frac{2}{7} \div \frac{2}{7}$

13. $\frac{3}{7} \div \frac{7}{3}$

14. $\frac{11}{12} \div \frac{1}{5}$

15. $\frac{4}{5} \div 1$

16. $1 \div \frac{3}{7}$

17. $\frac{3}{11} \div 4$

18. $2 \div \frac{7}{8}$

19. $1 \div \frac{7}{27}$

20. $\frac{9}{16} \div 1$

21. $0 \div \frac{3}{17}$

22. $0 \div \frac{5}{16}$

23. $\frac{18}{19} \div 0$

24. $\frac{24}{29} \div 0$

25. $8 \div \frac{4}{5}$

26. $16 \div \frac{8}{11}$

27. $\frac{7}{8} \div 4$

28. $\frac{5}{6} \div 12$

29. $\frac{9}{16} \div \frac{3}{4}$

30. $\frac{3}{4} \div \frac{9}{16}$

31. $3\frac{1}{4} \div 2\frac{1}{4}$

32. $2\frac{2}{3} \div 4\frac{1}{3}$

33. $6\frac{2}{5} \div 3\frac{1}{5}$

34. $9\frac{1}{3} \div 3\frac{1}{9}$

35. $6000 \div \frac{6}{5}$

36. $8000 \div \frac{4}{7}$

37. $\dfrac{\frac{4}{5}}{200}$

38. $\dfrac{\frac{5}{9}}{100}$

39. $\dfrac{\frac{5}{8}}{\frac{25}{7}}$

40. $\dfrac{\frac{3}{16}}{\frac{5}{8}}$

Mixed Practice *Multiply or divide.*

41. $3\frac{1}{5} \div \frac{1}{5}$

42. $4\frac{3}{4} \div \frac{1}{4}$

43. $2\frac{1}{3} \times \frac{1}{6}$

44. $6\frac{1}{2} \times \frac{1}{3}$

45. $5\frac{1}{4} \div 2\frac{5}{8}$

46. $1\frac{2}{9} \div 4\frac{1}{3}$

47. $5 \div 1\frac{1}{4}$

48. $7 \div 1\frac{2}{5}$

49. $5\frac{2}{3} \div 2\frac{1}{4}$

50. $14\frac{2}{3} \div 3\frac{1}{2}$

51. $\frac{7}{2} \div 3\frac{1}{2}$

52. $\frac{16}{3} \div 5\frac{1}{3}$

53. $\frac{13}{25} \times 2\frac{1}{3}$

54. $\frac{11}{20} \times 4\frac{1}{2}$

55. $3\frac{3}{4} \div 9$

56. $5\frac{5}{6} \div 7$

57. $\dfrac{5}{3\frac{1}{6}}$

58. $\dfrac{8}{2\frac{1}{2}}$

59. $\dfrac{0}{4\frac{3}{8}}$

60. $\dfrac{5\frac{2}{5}}{0}$

61. $\dfrac{\frac{7}{12}}{3\frac{2}{3}}$

62. $\dfrac{\frac{9}{10}}{3\frac{3}{5}}$

63. $4\frac{2}{5} \times 2\frac{8}{11}$

64. $4\frac{2}{3} \times 5\frac{1}{7}$

Review Example 6. Then find the value of x in each of the following.

65. $x \div \frac{4}{3} = \frac{21}{20}$

66. $x \div \frac{2}{5} = \frac{15}{16}$

67. $x \div \frac{10}{7} = \frac{21}{100}$

68. $x \div \frac{11}{6} = \frac{54}{121}$

Applications *Answer each question.*

69. ***Leather Factory*** A leather factory in Morocco tans leather. In order to make the leather soft, it has to soak in a vat of uric acid and other ingredients. The main holding tank holds $60\frac{3}{4}$ litres of the tanning mixture. If the mixture is distributed evenly into nine vats of equal size for the different coloured leathers, how much will each vat hold?

70. ***Marine Biology*** A specially protected stretch of beach bordering the Great Barrier Reef in Australia is used for marine biology and ecological research. The beach, which is $7\frac{1}{2}$ kilometres long, has been broken up into 20 equal segments for comparison purposes. How long is each segment of the beach?

71. ***Vehicle Travel*** Bruce drove in a snowstorm to get to his favourite mountain to do some snowboarding. He travelled 125 kilometres in $3\frac{1}{3}$ hours. What was his average speed (in kilometres per hour)?

72. ***Vehicle Travel*** Roberto drove his truck to Cedarville, a distance of 200 kilometres, in $4\frac{1}{6}$ hours. What was his average speed (in kilometres per hour)?

73. ***Cooking*** The school cafeteria is making hamburgers for the annual Senior Day Festival. The cooks have decided that because hamburger shrinks on the grill, they will allow $\frac{2}{3}$ pound of meat for each student. If the kitchen has $38\frac{2}{3}$ pounds of meat, how many students will be fed?

74. ***Making Costumes*** Costumes are needed for the junior high school's "Wizard of Oz" performance. Each costume requires $4\frac{1}{3}$ metres of fabric and $151\frac{2}{3}$ metres are available. How many costumes can be made?

75. ***Cooking*** A coffee pot that holds 150 cups of coffee is being used at a company meeting. Each large Styrofoam cup holds $1\frac{1}{2}$ cups of coffee. How many large Styrofoam cups can be filled?

76. ***Medicine Dosage*** A small bottle of eye drops contains 16 millilitres. If the recommended use is $\frac{2}{3}$ millilitre, how many times can a person use the drops before the bottle is empty?

77. ***Time Capsule*** In 1907, a time capsule was placed behind a steel wall measuring $4\frac{3}{4}$ inches thick. On December 22, 2007, a special drill was used to bore through the wall and extricate the time capsule. The drill could move only $\frac{5}{6}$ inch at a time. How many drill attempts did it take to reach the other side of the steel wall?

78. ***Ink Production*** Imagination Ink supplies different coloured inks for highlighter pens. Vat 1 has yellow ink, holds 150 litres, and is $\frac{4}{5}$ full. Vat 2 has green ink, holds 50 litres, and is $\frac{5}{8}$ full. One litre of ink will fill 300 pens. How many pens can be filled with the existing ink from Vats 1 and 2?

To Think About *When multiplying or dividing mixed numbers, it is wise to estimate your answer by rounding each mixed number to the nearest whole number.*

79. Estimate your answer to $14\frac{2}{3} \div 5\frac{1}{6}$ by rounding each mixed number to the nearest whole number. Then find the exact answer. How close was your estimate?

80. Estimate your answer to $18\frac{1}{4} \times 27\frac{1}{2}$ by rounding each mixed number to the nearest whole number. Then find the exact answer. How close was your estimate?

Quick Quiz 5 Divide.

1. $\frac{15}{24} \div \frac{5}{6}$

2. $6\frac{1}{3} \div 2\frac{5}{12}$

3. $7\frac{3}{4} \div 4$

4. **Concept Check** Explain how you would divide the whole number 7 by the mixed number $3\frac{3}{5}$.

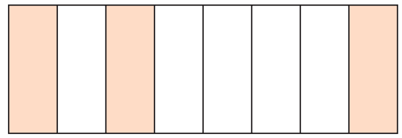
How Am I Doing? Sections 1–5

1. ____
2. ____
3. ____
4. ____
5. ____
6. ____
7. ____
8. ____
9. ____
10. ____
11. ____
12. ____
13. ____
14. ____
15. ____
16. ____
17. ____
18. ____
19. ____
20. ____

How are you doing with your homework assignments in Sections 1 to 5? Do you feel you have mastered the material so far? Do you understand the concepts you have covered? Before you go further, take some time to do each of the following problems.

1

1. Use a fraction to represent the shaded part of the object.



2. Frederich University had 3500 students from inside the province, 2600 students from outside the province but inside the country, and 800 students from outside the country. Write a fraction that describes the part of the student body from outside the country. Reduce the fraction.

3. An inspector checked 124 CD players. Of these, 5 were defective. Write a fraction that describes the part that was defective.

2

Reduce each fraction.

4. $\frac{3}{18}$ **5.** $\frac{13}{39}$ **6.** $\frac{16}{112}$ **7.** $\frac{175}{200}$ **8.** $\frac{44}{121}$

3

Change to an improper fraction.

9. $3\frac{2}{3}$ **10.** $15\frac{1}{3}$

Change to a mixed number.

11. $\frac{81}{4}$ **12.** $\frac{29}{5}$ **13.** $\frac{36}{17}$

4

Multiply.

14. $\frac{5}{11} \times \frac{1}{4}$ **15.** $\frac{3}{7} \times \frac{14}{9}$ **16.** $3\frac{1}{3} \times 5\frac{1}{3}$

5

Divide.

17. $\frac{3}{7} \div \frac{3}{7}$ **18.** $\frac{7}{16} \div \frac{7}{8}$ **19.** $6\frac{4}{7} \div 1\frac{5}{21}$ **20.** $12 \div \frac{4}{7}$

Now turn to the end of the module for the answer to each of these problems. Each answer also includes a reference to the objective in which the problem is first taught. If you missed any of these problems, you should stop and review the Examples and Practice Problems in the referenced objective. A little review now will help you master the material in the upcoming sections.

How Am I Doing? Test on Sections 1–5

Solve. Make sure all fractions are simplified in the final answer.

1. Norah answered 33 out of 40 questions correctly on her chemistry exam. Write a fraction that describes the part of the exam she answered correctly.

2. Carlos inspected the boxes that were shipped from the central warehouse. He found that 340 were the correct weight and 112 were not. Write a fraction that describes what part of the total number of the boxes were at the correct weight.

Reduce each fraction.

3. $\frac{19}{38}$ **4.** $\frac{40}{56}$ **5.** $\frac{24}{66}$

6. $\frac{125}{155}$ **7.** $\frac{50}{140}$ **8.** $\frac{84}{36}$

Change each mixed number to an improper fraction.

9. $12\frac{2}{3}$ **10.** $4\frac{1}{8}$

Change each improper fraction to a mixed number.

11. $\frac{45}{7}$ **12.** $\frac{75}{9}$

Multiply.

13. $\frac{3}{8} \times \frac{7}{11}$ **14.** $\frac{35}{16} \times \frac{4}{5}$

15. $18 \times \frac{5}{6}$ **16.** $\frac{3}{8} \times 44$

17. $2\frac{1}{3} \times 5\frac{3}{4}$ **18.** $24 \times 3\frac{1}{3}$

Divide.

19. $\frac{4}{7} \div \frac{3}{4}$ **20.** $\frac{8}{9} \div \frac{1}{6}$

21. $5\frac{1}{4} \div \frac{3}{4}$ **22.** $5\frac{3}{5} \div 2\frac{1}{3}$

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________
9. ____________
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28. ____
29. ____
30. ____
31. ____
32. ____
33. ____
34. ____
35. ____
36. ____
37. ____
38. ____
39. ____

Mixed Practice

Perform the indicated operations. Simplify your answers.

23. $2\frac{1}{4} \times 3\frac{1}{2}$

24. $6 \times 2\frac{1}{3}$

25. $5 \div 1\frac{7}{8}$

26. $5\frac{3}{4} \div 2$

27. $\frac{13}{20} \div \frac{4}{5}$

28. $\frac{4}{7} \div 8$

29. $\frac{9}{22} \times \frac{11}{16}$

30. $\frac{14}{25} \times \frac{65}{42}$

Solve. Simplify your answer.

▲ **31.** A garden measures $5\frac{1}{4}$ metres by $8\frac{3}{4}$ metres. What is the area of the garden in square metres?

32. A recipe for two loaves of bread calls for $2\frac{2}{3}$ cups of flour. Lexi wants to make $1\frac{1}{2}$ times as much bread. How many cups of flour will she need?

33. Lisa drove $62\frac{1}{2}$ kilometres to visit a friend. Three-fourths of her trip was on the highway. How many kilometres did she drive on the highway?

34. The butcher prepared $12\frac{3}{8}$ kilograms of lean ground round. He placed it in packages that held $\frac{3}{4}$ of a kilogram. How many full packages did he have? How much lean ground round was left over?

35. The college computer centre has 136 computers. Samuel found that $\frac{3}{8}$ of them have Windows XP installed on them. How many computers have Windows XP installed on them?

36. The average household uses 310 000 litres of water each year. About $\frac{3}{10}$ of this amount is used for showers and baths. How many litres of water are used each year for showers and baths in an average household?

37. Yung Kim was paid \$132 last week at his part-time job. He was paid $\$8\frac{1}{4}$ per hour. How many hours did he work last week?

38. The Outdoor Shop is making some custom tents that are very light but totally waterproof. Each tent requires $8\frac{1}{4}$ metres of cloth. How many tents can be made from $56\frac{1}{2}$ metres of cloth? How much cloth will be left over?

39. A container of vanilla-flavoured syrup holds $32\frac{4}{5}$ centilitres. Nate uses $\frac{4}{5}$ centilitre every morning in his coffee. How many days will it take Nate to use up the container?

Section 6 The Least Common Denominator and Creating Equivalent Fractions

Student Learning Objectives

After studying this section, you will be able to:

1. Find the least common multiple (LCM) of two numbers.
2. Find the least common denominator (LCD) given two or three fractions.
3. Create equivalent fractions with a least common denominator.

1 Finding the Least Common Multiple (LCM) of Two Numbers

The idea of a multiple of a number is fairly straightforward.

The **multiples** of a number are the products of that number and the numbers 1, 2, 3, 4, 5, 6, 7, . . .

For example, the multiples of 4 are	4, 8, 12, 16, 20, 24, 28, . . .
The multiples of 5 are	5, 10, 15, 20, 25, 30, 35, . . .

The **least common multiple,** or **LCM,** of two natural numbers is the smallest number that is a multiple of both.

EXAMPLE 1 Find the least common multiple of 10 and 12.

Solution

The multiples of 10 are	10,	20,	30,	40,	50,	60,	70, . . .
The multiples of 12 are	12,	24,	36,	48,	60,	72,	84, . . .

The first multiple that appears on both lists is the least common multiple. Thus the number 60 is the least common multiple of 10 and 12.

Practice Problem 1 Find the least common multiple of 14 and 21.

NOTE TO STUDENT: Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.

EXAMPLE 2 Find the least common multiple of 6 and 8.

Solution

The multiples of 6 are	6,	12,	18,	24,	30,	36,	42, . . .
The multiples of 8 are	8,	16,	24,	32,	40,	48,	56, . . .

The first multiple that appears on both lists is the least common multiple. Thus the number 24 is the least common multiple of 6 and 8.

Practice Problem 2 Find the least common multiple of 10 and 15.

Now, of course, we can do the problem immediately if the larger number is a multiple of the smaller number. In such cases the larger number is the least common multiple.

EXAMPLE 3 Find the least common multiple of 7 and 35.

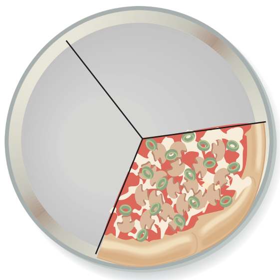
Solution Because $7 \times 5 = 35$, 35 is a multiple of 7.

So we can state immediately that the least common multiple of 7 and 35 is 35.

Practice Problem 3 Find the least common multiple of 6 and 54.

2 Finding the Least Common Denominator (LCD) Given Two or Three Fractions

We need some way to determine which of two fractions is larger. Suppose that Marcia and Melissa each have some leftover pizza.

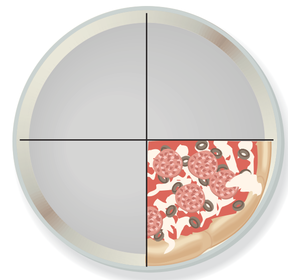


Marcia's Pizza

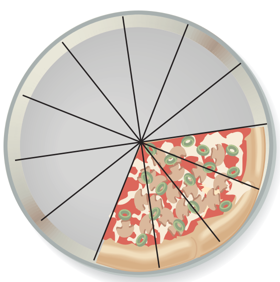
$\frac{1}{3}$ of a pizza left

Melissa's Pizza

$\frac{1}{4}$ of a pizza left



Who has more pizza left? How much more? Comparing the amounts of pizza left would be easy if each pizza had been cut into equal-sized pieces. If the original pizzas had each been cut into 12 pieces, we would be able to see that Marcia had $\frac{1}{12}$ of a pizza more than Melissa had.

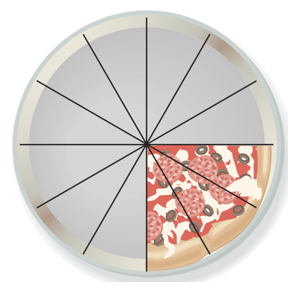


Marcia's Pizza

$\left(\begin{array}{c}\text{We know that} \\ \frac{4}{12} = \frac{1}{3} \text{ by reducing.}\end{array}\right)$

Melissa's Pizza

$\left(\begin{array}{c}\text{We know that} \\ \frac{3}{12} = \frac{1}{4} \text{ by reducing.}\end{array}\right)$



The denominator 12 appears in the fractions $\frac{4}{12}$ and $\frac{3}{12}$. We call the smallest denominator that allows us to compare fractions directly the *least common denominator,* abbreviated LCD. The number 12 is the least common denominator for the fractions $\frac{1}{3}$ and $\frac{1}{4}$.

Notice that 12 is the least common multiple of 3 and 4.

LEAST COMMON DENOMINATOR

The **least common denominator (LCD)** of two or more fractions is the smallest number that can be divided evenly by each of the fractions' denominators.

How does this relate to least common multiples? The LCD of two fractions is the least common multiple of the two denominators.

In some problems you may be able to guess the LCD quite quickly. With practice, you can often find the LCD mentally. For example, you now know that if the denominators of two fractions are 3 and 4, the LCD is 12. For the fractions $\frac{1}{2}$ and $\frac{1}{4}$, the LCD is 4; for the fractions $\frac{1}{3}$ and $\frac{1}{6}$, the LCD is 6. We can see that if the denominator of one fraction divides without remainder into the denominator of another, the LCD of the two fractions is the larger of the denominators.

EXAMPLE 4 Determine the LCD for each pair of fractions.

(a) $\frac{7}{15}$ and $\frac{4}{5}$ **(b)** $\frac{2}{3}$ and $\frac{5}{27}$

Solution

(a) Since 5 can be divided into 15, the LCD of $\frac{7}{15}$ and $\frac{4}{5}$ is 15. (Notice that the least common multiple of 5 and 15 is 15.)

(b) Since 3 can be divided into 27, the LCD of $\frac{2}{3}$ and $\frac{5}{27}$ is 27. (Notice that the least common multiple of 3 and 27 is 27.)

Practice Problem 4 Determine the LCD for each pair of fractions.

(a) $\frac{3}{4}$ and $\frac{11}{12}$ **(b)** $\frac{1}{7}$ and $\frac{8}{35}$

NOTE TO STUDENT: *Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.*

In a few cases, the LCD is the product of the two denominators.

EXAMPLE 5 Find the LCD for $\frac{1}{4}$ and $\frac{3}{5}$.

Solution We see that $4 \times 5 = 20$. Also, 20 is the *smallest* number that can be divided without remainder by 4 and by 5. We know this because the least common multiple of 4 and 5 is 20. So the LCD $= 20$.

Practice Problem 5 Find the LCD for $\frac{3}{7}$ and $\frac{5}{6}$.

In cases where the LCD is not obvious, the following procedure will help us find the LCD.

THREE-STEP PROCEDURE FOR FINDING THE LEAST COMMON DENOMINATOR

1. Write each denominator as the product of prime factors.

2. List all the prime factors that appear in either product.

3. Form a product of those prime factors, using each factor the greatest number of times it appears in any one denominator.

EXAMPLE 6 Find the LCD by the three-step procedure.

(a) $\frac{5}{6}$ and $\frac{4}{15}$ **(b)** $\frac{7}{18}$ and $\frac{7}{30}$ **(c)** $\frac{10}{27}$ and $\frac{5}{18}$

Solution

(a) Step 1 Write each denominator as a product of prime factors.

$$6 = 2 \times 3 \qquad 15 = 5 \times 3$$

Step 2 The LCD will contain the factors 2, 3, and 5.

$$6 = 2 \times 3 \qquad 15 = 5 \times 3$$

Step 3 LCD $= 2 \times 3 \times 5$ We form a product.

$$= 30$$

(b) Step 1 Write each denominator as a product of prime factors.

$$18 = 2 \times 9 = 2 \times 3 \times 3$$
$$30 = 3 \times 10 = 2 \times 3 \times 5$$

Step 2 The LCD will be a product containing 2, 3, and 5.

Step 3 The LCD will contain the factor 3 twice since it occurs twice in the denominator 18.

Factor 3 occurs twice in one denominator.

$$18 = 2 \times 3 \times 3$$
$$\text{LCD} = 2 \times 3 \times 3 \times 5 = 90$$

(c) Write each denominator as a product of prime factors.

$$27 = 3 \times 3 \times 3 \qquad 18 = 3 \times 3 \times 2$$

Factor 3 occurs three times.

The LCD will contain the factor 2 once but the factor 3 three times.

$$\text{LCD} = 2 \times 3 \times 3 \times 3 = 54$$

Practice Problem 6 Find the LCD for each pair of fractions.

(a) $\frac{3}{14}$ and $\frac{1}{10}$ **(b)** $\frac{1}{15}$ and $\frac{7}{50}$ **(c)** $\frac{3}{16}$ and $\frac{5}{12}$

A similar procedure can be used for three fractions.

EXAMPLE 7 Find the LCD of $\frac{7}{12}, \frac{1}{15},$ and $\frac{11}{30}$.

Solution

$$\begin{aligned} 12 &= 2 \times 2 \times 3 \\ 15 &= \phantom{2 \times 2 \times {}} 3 \times 5 \\ 30 &= \phantom{2 \times {}} 2 \times 3 \times 5 \\ \text{LCD} &= 2 \times 2 \times 3 \times 5 \\ &= 60 \end{aligned}$$

Practice Problem 7 Find the LCD of $\frac{3}{49}$, $\frac{5}{21}$, and $\frac{6}{7}$.

3 Creating Equivalent Fractions with a Least Common Denominator

In Section 7, we will discuss how to add fractions. We cannot add fractions that have different denominators. To change denominators, we must (1) find the LCD and (2) build up the addends—the fractions being added—into equivalent fractions that have the LCD as the denominator. We know now how to find the LCD. Let's look at how we build fractions. We know, for example, that

$$\frac{1}{2} = \frac{2}{4} = \frac{50}{100} \qquad \frac{1}{4} = \frac{25}{100} \qquad \text{and} \qquad \frac{3}{4} = \frac{75}{100}.$$

In these cases, we have mentally multiplied the given fraction by 1, in the form of a certain number, c, in the numerator and that same number, c, in the denominator.

$$\frac{1}{2} \times \boxed{\frac{c}{c}} = \frac{2}{4} \qquad \text{Here } c = 2, \frac{2}{2} = 1.$$

$$\frac{1}{2} \times \boxed{\frac{c}{c}} = \frac{50}{100} \qquad \text{Here } c = 50, \frac{50}{50} = 1.$$

This property is called the *building fraction property*.

BUILDING FRACTION PROPERTY

For whole numbers a, b, and c where $b \neq 0$, $c \neq 0$,

$$\frac{a}{b} = \frac{a}{b} \times 1 = \frac{a}{b} \times \boxed{\frac{c}{c}} = \frac{a \times c}{b \times c}.$$

EXAMPLE 8 Build each fraction to an equivalent fraction with the given LCD.

(a) $\frac{3}{4}$, LCD $= 28$ **(b)** $\frac{4}{5}$, LCD $= 45$ **(c)** $\frac{1}{3}$ and $\frac{4}{5}$, LCD $= 15$

Solution

(a) $\frac{3}{4} \times \boxed{\frac{c}{c}} = \frac{?}{28}$ We know that $4 \times 7 = 28$, so the value c that we multiply numerator and denominator by is 7.

$$\frac{3}{4} \times \frac{7}{7} = \frac{21}{28}$$

(b) $\frac{4}{5} \times \boxed{\frac{c}{c}} = \frac{?}{45}$ We know that $5 \times 9 = 45$, so $c = 9$.

$$\frac{4}{5} \times \frac{9}{9} = \frac{36}{45}$$

(c) $\dfrac{1}{3} = \dfrac{?}{15}$ We know that $3 \times 5 = 15$, so we multiply numerator and denominator by 5.

$$\frac{1}{3} \times \boxed{\frac{5}{5}} = \frac{5}{15}$$

$\dfrac{4}{5} = \dfrac{?}{15}$ We know that $5 \times 3 = 15$, so we multiply numerator and denominator by 3.

$$\frac{4}{5} \times \boxed{\frac{3}{3}} = \frac{12}{15}$$

Thus $\dfrac{1}{3} = \dfrac{5}{15}$ and $\dfrac{4}{5} = \dfrac{12}{15}$.

***NOTE TO STUDENT:** Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.*

Practice Problem 8 Build each fraction to an equivalent fraction with the LCD.

(a) $\dfrac{3}{5}$, LCD $= 40$ **(b)** $\dfrac{7}{11}$, LCD $= 44$ **(c)** $\dfrac{2}{7}$ and $\dfrac{3}{4}$, LCD $= 28$

EXAMPLE 9

(a) Find the LCD of $\dfrac{1}{32}$ and $\dfrac{7}{48}$.

(b) Build the fractions to equivalent fractions that have the LCD as their denominators.

Solution

(a) First we find the prime factors of 32 and 48.

$$32 = 2 \times 2 \times 2 \times 2 \times 2$$
$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Thus the LCD will require a factor of 2 five times and a factor of 3 one time.

$$\text{LCD} = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$$

(b) $\dfrac{1}{32} = \dfrac{?}{96}$ Since $32 \times 3 = 96$ we multiply by the fraction $\dfrac{3}{3}$.

$$\frac{1}{32} = \frac{1}{32} \times \boxed{\frac{3}{3}} = \frac{3}{96}$$

$\dfrac{7}{48} = \dfrac{?}{96}$ Since $48 \times 2 = 96$, we multiply by the fraction $\dfrac{2}{2}$.

$$\frac{7}{48} = \frac{7}{48} \times \boxed{\frac{2}{2}} = \frac{14}{96}$$

Practice Problem 9

(a) Find the LCD of $\dfrac{3}{20}$ and $\dfrac{11}{15}$.

(b) Build the fractions to equivalent fractions that have the LCD as their denominators.

EXAMPLE 10

(a) Find the LCD of $\frac{2}{125}$ and $\frac{8}{75}$.

(b) Build the fractions to equivalent fractions that have the LCD as their denominators.

Solution

(a) First we find the prime factors of 125 and 75.

$$125 = 5 \times 5 \times 5$$
$$75 = 5 \times 5 \times 3$$

Thus the LCD will require a factor of 5 three times and a factor of 3 one time.

$$\text{LCD} = 5 \times 5 \times 5 \times 3 = 375$$

(b) $\frac{2}{125} = \frac{?}{375}$ Since $125 \times 3 = 375$, we multiply by the fraction $\frac{3}{3}$.

$$\frac{2}{125} = \frac{2}{125} \times \boxed{\frac{3}{3}} = \frac{6}{375}$$

$\frac{8}{75} = \frac{?}{375}$ Since $75 \times 5 = 375$, we multiply by the fraction $\frac{5}{5}$.

$$\frac{8}{75} = \frac{8}{75} \times \boxed{\frac{5}{5}} = \frac{40}{375}$$

Practice Problem 10

(a) Find the LCD of $\frac{5}{64}$ and $\frac{3}{80}$.

(b) Build the fractions to equivalent fractions that have the LCD as their denominators.

NOTE TO STUDENT: Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.

SECTION 6 EXERCISES

Find the least common multiple (LCM) for each pair of numbers.

1. 8 and 12 **2.** 6 and 9 **3.** 20 and 50 **4.** 22 and 55 **5.** 12 and 15

6. 18 and 30 **7.** 10 and 15 **8.** 8 and 60 **9.** 21 and 49 **10.** 25 and 35

Find the LCD for each pair of fractions.

11. $\frac{1}{5}$ and $\frac{3}{10}$ **12.** $\frac{3}{8}$ and $\frac{5}{16}$ **13.** $\frac{3}{7}$ and $\frac{1}{4}$ **14.** $\frac{5}{6}$ and $\frac{3}{5}$ **15.** $\frac{2}{5}$ and $\frac{3}{7}$

16. $\frac{1}{16}$ and $\frac{2}{3}$ **17.** $\frac{1}{6}$ and $\frac{5}{9}$ **18.** $\frac{1}{4}$ and $\frac{3}{14}$ **19.** $\frac{7}{12}$ and $\frac{14}{15}$ **20.** $\frac{7}{15}$ and $\frac{9}{25}$

21. $\frac{7}{32}$ and $\frac{3}{4}$ **22.** $\frac{2}{11}$ and $\frac{1}{44}$ **23.** $\frac{5}{10}$ and $\frac{11}{45}$ **24.** $\frac{13}{20}$ and $\frac{17}{30}$ **25.** $\frac{7}{16}$ and $\frac{17}{80}$

26. $\frac{5}{6}$ and $\frac{19}{30}$ **27.** $\frac{5}{21}$ and $\frac{8}{35}$ **28.** $\frac{1}{20}$ and $\frac{5}{70}$ **29.** $\frac{11}{24}$ and $\frac{7}{30}$ **30.** $\frac{23}{30}$ and $\frac{37}{50}$

Find the LCD for each set of three fractions.

31. $\frac{2}{3}, \frac{1}{2}, \frac{5}{6}$ **32.** $\frac{1}{5}, \frac{1}{3}, \frac{7}{10}$ **33.** $\frac{1}{4}, \frac{11}{12}, \frac{5}{6}$ **34.** $\frac{21}{48}, \frac{1}{12}, \frac{3}{8}$

35. $\frac{5}{11}, \frac{7}{12}, \frac{1}{6}$ **36.** $\frac{11}{16}, \frac{3}{20}, \frac{2}{5}$ **37.** $\frac{7}{12}, \frac{1}{21}, \frac{3}{14}$ **38.** $\frac{1}{30}, \frac{3}{40}, \frac{7}{8}$

39. $\frac{7}{15}, \frac{11}{12}, \frac{7}{8}$ **40.** $\frac{5}{36}, \frac{2}{48}, \frac{1}{24}$

Build each fraction to an equivalent fraction with the specified denominator. State the numerator.

41. $\frac{1}{3} = \frac{?}{9}$

42. $\frac{1}{5} = \frac{?}{35}$

43. $\frac{5}{7} = \frac{?}{49}$

44. $\frac{7}{9} = \frac{?}{81}$

45. $\frac{4}{11} = \frac{?}{55}$

46. $\frac{2}{13} = \frac{?}{39}$

47. $\frac{5}{12} = \frac{?}{96}$

48. $\frac{3}{50} = \frac{?}{100}$

49. $\frac{8}{9} = \frac{?}{108}$

50. $\frac{6}{7} = \frac{?}{147}$

51. $\frac{7}{20} = \frac{?}{180}$

52. $\frac{3}{25} = \frac{?}{175}$

The LCD of each pair of fractions is listed. Build each fraction to an equivalent fraction that has the LCD as the denominator.

53. LCD $= 36, \frac{7}{12}$ and $\frac{5}{9}$

54. LCD $= 20, \frac{9}{10}$ and $\frac{3}{4}$

55. LCD $= 80, \frac{5}{16}$ and $\frac{17}{20}$

56. LCD $= 72, \frac{5}{24}$ and $\frac{7}{36}$

57. LCD $= 20, \frac{9}{10}$ and $\frac{19}{20}$

58. LCD $= 240, \frac{13}{30}$ and $\frac{41}{80}$

Find the LCD. Build the fractions to equivalent fractions having the LCD as the denominator.

59. $\frac{2}{5}$ and $\frac{9}{35}$

60. $\frac{7}{9}$ and $\frac{35}{54}$

61. $\frac{5}{24}$ and $\frac{3}{8}$

62. $\frac{19}{42}$ and $\frac{6}{7}$

63. $\frac{8}{15}$ and $\frac{1}{6}$

64. $\frac{19}{20}$ and $\frac{7}{8}$

65. $\frac{4}{15}$ and $\frac{5}{12}$

66. $\frac{9}{10}$ and $\frac{3}{25}$

67. $\frac{5}{18}, \frac{11}{36}, \frac{7}{12}$

68. $\frac{1}{30}, \frac{7}{15}, \frac{1}{45}$

69. $\frac{3}{56}, \frac{7}{8}, \frac{5}{7}$

70. $\frac{5}{9}, \frac{1}{6}, \frac{3}{54}$

71. $\frac{5}{63}, \frac{4}{21}, \frac{8}{9}$

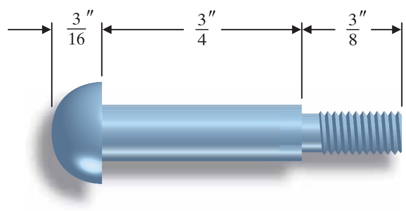
72. $\frac{3}{8}, \frac{5}{14}, \frac{13}{16}$

Applications

73. ***Door Repair*** Suppose that you wish to compare the lengths of the three portions of the given stainless steel bolt that came out of a door.

(a) What is the LCD for the three fractions?

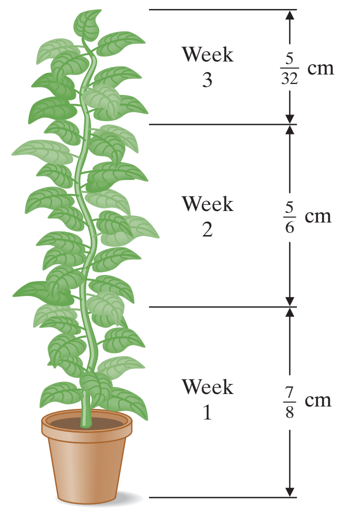
(b) Build each fraction to an equivalent fraction that has the LCD as a denominator.



74. ***Plant Growth*** Suppose that you want to prepare a report on the growth of a plant. The total height of the plant in the pot is recorded for each week of a three-week experiment.

(a) What is the LCD for the three fractions?

(b) Build each fraction to an equivalent fraction that has the LCD for a denominator.



Quick Quiz 6

1. Find the least common denominator of

$$\frac{5}{6} \text{ and } \frac{5}{21}$$

2. Find the least common denominator of

$$\frac{27}{28}, \frac{3}{4}, \frac{19}{20}$$

3. Build the fraction to an equivalent fraction with the specified denominator.

$$\frac{7}{26} = \frac{?}{78}$$

4. **Concept Check** Explain how you would find the least common denominator of the fractions $\frac{5}{6}$, $\frac{11}{14}$, and $\frac{2}{15}$.

SECTION 7 ADDING AND SUBTRACTING FRACTIONS

Student Learning Objectives

After studying this section, you will be able to:

1. **Add and subtract fractions with a common denominator.**
2. **Add and subtract fractions with different denominators.**

1 Adding and Subtracting Fractions with a Common Denominator

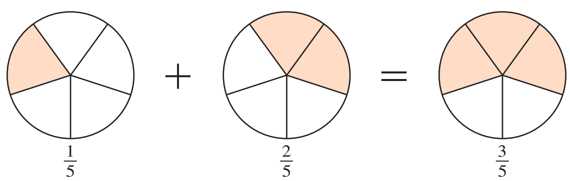
You must have common denominators (denominators that are alike) to add or subtract fractions.

If your problem has fractions without a common denominator or if it has mixed numbers, you must use what you already know about changing the form of each fraction (how the fraction looks). Only after all the fractions have a common denominator can you add or subtract.

An important distinction: You must have common denominators to add or subtract fractions, but you need not have common denominators to multiply or divide fractions.

To add two fractions that have the same denominator, add the numerators and write the sum over the common denominator.

To illustrate we use $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$. The figure shows that $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$.



EXAMPLE 1 Add. $\frac{5}{13} + \frac{7}{13}$

Solution

$$\frac{5}{13} + \frac{7}{13} = \frac{12}{13}$$

Practice Problem 1 Add.

$$\frac{3}{17} + \frac{12}{17}$$

NOTE TO STUDENT: Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.

The answer may need to be reduced. Sometimes the answer may be written as a mixed number.

EXAMPLE 2 Add.

(a) $\frac{4}{9} + \frac{2}{9}$ **(b)** $\frac{5}{7} + \frac{6}{7}$

Solution

(a) $\frac{4}{9} + \frac{2}{9} = \frac{6}{9} = \frac{2}{3}$ **(b)** $\frac{5}{7} + \frac{6}{7} = \frac{11}{7}$ or $1\frac{4}{7}$

Practice Problem 2 Add.

(a) $\frac{1}{12} + \frac{5}{12}$ **(b)** $\frac{13}{15} + \frac{7}{15}$

A similar rule is followed for subtraction, except that the numerators are subtracted and the result placed over the common denominator. Be sure to reduce all answers when possible.

EXAMPLE 3 Subtract.

(a) $\frac{5}{13} - \frac{4}{13}$ **(b)** $\frac{17}{20} - \frac{3}{20}$

Solution

(a) $\frac{5}{13} - \frac{4}{13} = \frac{1}{13}$ **(b)** $\frac{17}{20} - \frac{3}{20} = \frac{14}{20} = \frac{7}{10}$

Practice Problem 3 Subtract.

(a) $\frac{5}{19} - \frac{2}{19}$ **(b)** $\frac{21}{25} - \frac{6}{25}$

2 Adding and Subtracting Fractions with Different Denominators

If the two fractions do not have a common denominator, we follow the procedure in Section 6: Find the LCD and then build each fraction so that its denominator is the LCD.

EXAMPLE 4 Add. $\frac{7}{12} + \frac{1}{4}$

Solution The LCD is 12. The fraction $\frac{7}{12}$ already has the least common denominator.

$$\begin{array}{rcl} \frac{7}{12} & = & \frac{7}{12} \\ + \frac{1}{4} \times \frac{3}{3} & = & + \frac{3}{12} \\ \hline & & \frac{10}{12} \end{array}$$

We will need to reduce this fraction. Then we will have

$$\frac{7}{12} + \frac{1}{4} = \frac{7}{12} + \frac{3}{12} = \frac{10}{12} = \frac{5}{6}.$$

It is very important to remember to reduce our final answer.

Practice Problem 4 Add.

$$\frac{2}{15} + \frac{1}{5}$$

EXAMPLE 5 Add. $\dfrac{7}{20} + \dfrac{4}{15}$

Solution LCD = 60.

$$\frac{7}{20} \times \frac{3}{3} = \frac{21}{60} \qquad \frac{4}{15} \times \frac{4}{4} = \frac{16}{60}$$

Thus

$$\frac{7}{20} + \frac{4}{15} = \frac{21}{60} + \frac{16}{60} = \frac{37}{60}$$

Practice Problem 5 Add.

$$\frac{5}{12} + \frac{5}{16}$$

A similar procedure holds for the addition of three or more fractions.

EXAMPLE 6 Add. $\dfrac{3}{8} + \dfrac{5}{6} + \dfrac{1}{4}$

Solution LCD = 24.

$$\frac{3}{8} \times \frac{3}{3} = \frac{9}{24} \qquad \frac{5}{6} \times \frac{4}{4} = \frac{20}{24} \qquad \frac{1}{4} \times \frac{6}{6} = \frac{6}{24}$$

$$\frac{3}{8} + \frac{5}{6} + \frac{1}{4} = \frac{9}{24} + \frac{20}{24} + \frac{6}{24} = \frac{35}{24} \quad \text{or} \quad 1\frac{11}{24}$$

Practice Problem 6 Add.

$$\frac{3}{16} + \frac{1}{8} + \frac{1}{12}$$

EXAMPLE 7 Subtract. $\dfrac{17}{25} - \dfrac{3}{35}$

Solution LCD = 175.

$$\frac{17}{25} \times \frac{7}{7} = \frac{119}{175} \qquad \frac{3}{35} \times \frac{5}{5} = \frac{15}{175}$$

Thus

$$\frac{17}{25} - \frac{3}{35} = \frac{119}{175} - \frac{15}{175} = \frac{104}{175}.$$

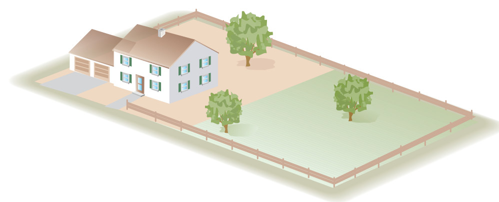
Practice Problem 7 Subtract.

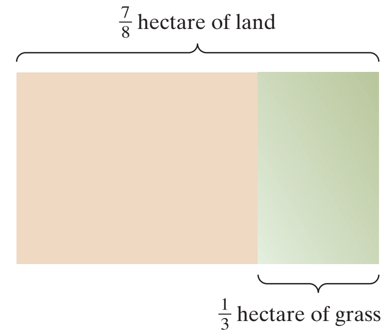
$$\frac{9}{48} - \frac{5}{32}$$

EXAMPLE 8 John and Stephanie have a house on $\frac{7}{8}$ hectare of land. They have $\frac{1}{3}$ hectare of land planted with grass. How much of the land is not planted with grass?

Solution

1. ***Understand the problem.*** Draw a picture.





We need to subtract. $\dfrac{7}{8} - \dfrac{1}{3}$

2. ***Solve and state the answer.*** The LCD is 24.

$$\frac{7}{8} \times \frac{3}{3} = \frac{21}{24} \qquad \frac{1}{3} \times \frac{8}{8} = \frac{8}{24}$$

$$\frac{7}{8} - \frac{1}{3} = \frac{21}{24} - \frac{8}{24} = \frac{13}{24}$$

We conclude that $\dfrac{13}{24}$ hectare of land is not planted with grass.

3. ***Check.*** The check is left to the student.

Practice Problem 8 Leon had $\frac{9}{10}$ litre of cleaning fluid in the garage. He used $\frac{1}{4}$ litre to clean the garage floor. How much cleaning fluid is left?

Some students may find Example 9 difficult. Read it slowly and carefully.

EXAMPLE 9 Find the value of x in the equation $x + \frac{5}{6} = \frac{9}{10}$. Reduce your answer.

Solution The LCD for the two fractions $\dfrac{5}{6}$ and $\dfrac{9}{10}$ is 30.

$$\frac{5}{6} \times \frac{5}{5} = \frac{25}{30} \qquad \frac{9}{10} \times \frac{3}{3} = \frac{27}{30}$$

Thus we can write the equation in the equivalent form.

$$x + \frac{25}{30} = \frac{27}{30}$$

The denominators are the same. Look at the numerators. We must add 2 to 25 to get 27.

$$\frac{2}{30} + \frac{25}{30} = \frac{27}{30}$$

So $x = \frac{2}{30}$ and we reduce the fraction to obtain $x = \frac{1}{15}$.

Practice Problem 9 Find the value of x in the equation $x + \frac{3}{10} = \frac{23}{25}$.

ALTERNATIVE METHOD: Multiply the Denominators as a Common Denominator In all the problems in this section so far, we have combined two fractions by first finding the least common denominator. However, there is an alternative approach. You are only required to find a common denominator, not necessarily the least common denominator. One way to quickly find a common denominator of two fractions is to multiply the two denominators. However, if you use this method, the numbers will usually be larger and you will usually need to simplify the fraction in your final answer.

EXAMPLE 10 Add $\frac{11}{12} + \frac{13}{30}$ by using the product of the two denominators as a common denominator.

Solution Using this method we just multiply the numerator and denominator of each fraction by the denominator of the other fraction. Thus no steps are needed to determine what to multiply by.

$$\frac{11}{12} \times \frac{30}{30} = \frac{330}{360} \qquad \frac{13}{30} \times \frac{12}{12} = \frac{156}{360}$$

Thus $\frac{11}{12} + \frac{13}{30} = \frac{330}{360} + \frac{156}{360} = \frac{486}{360}$

We must reduce the fraction: $\frac{486}{360} = \frac{27}{20}$ or $1\frac{7}{20}$

Practice Problem 10 Add $\frac{15}{16} + \frac{3}{40}$ by using the product of the two denominators as a common denominator.

NOTE TO STUDENT:* *Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.

Some students find this alternative method helpful because you do not have to find the LCD or the number each fraction must be multiplied by. Other students find this alternative method more difficult because of errors encountered when working with large numbers or in reducing the final answer. You are encouraged to try a couple of the homework exercises by this method and make up your own mind.

Section 7 Exercises

Add or subtract. Simplify all answers.

1. $\frac{5}{9} + \frac{2}{9}$

2. $\frac{5}{8} + \frac{2}{8}$

3. $\frac{7}{18} + \frac{15}{18}$

4. $\frac{11}{25} + \frac{17}{25}$

5. $\frac{19}{20} - \frac{11}{20}$

6. $\frac{17}{30} - \frac{7}{30}$

7. $\frac{53}{88} - \frac{19}{88}$

8. $\frac{103}{110} - \frac{3}{110}$

Add or subtract. Simplify all answers.

9. $\frac{1}{3} + \frac{1}{2}$

10. $\frac{1}{4} + \frac{1}{3}$

11. $\frac{3}{10} + \frac{3}{20}$

12. $\frac{4}{9} + \frac{1}{6}$

13. $\frac{1}{8} + \frac{3}{4}$

14. $\frac{5}{16} + \frac{1}{2}$

15. $\frac{4}{5} + \frac{7}{20}$

16. $\frac{2}{3} + \frac{4}{7}$

17. $\frac{3}{10} + \frac{7}{100}$

18. $\frac{13}{100} + \frac{7}{10}$

19. $\frac{3}{10} + \frac{1}{6}$

20. $\frac{8}{15} + \frac{3}{10}$

21. $\frac{7}{8} + \frac{5}{12}$

22. $\frac{5}{6} + \frac{7}{8}$

23. $\frac{3}{8} + \frac{3}{10}$

24. $\frac{12}{35} + \frac{1}{10}$

25. $\frac{29}{18} - \frac{5}{9}$

26. $\frac{37}{20} - \frac{2}{5}$

27. $\frac{3}{7} - \frac{9}{21}$

28. $\frac{7}{8} - \frac{5}{6}$

29. $\frac{5}{9} - \frac{5}{36}$

30. $\frac{9}{10} - \frac{1}{15}$

31. $\frac{5}{12} - \frac{7}{30}$

32. $\frac{9}{24} - \frac{3}{8}$

33. $\frac{11}{12} - \frac{2}{3}$

34. $\frac{7}{10} - \frac{2}{5}$

35. $\frac{17}{21} - \frac{1}{7}$

36. $\frac{20}{25} - \frac{4}{5}$

37. $\frac{5}{12} - \frac{7}{18}$

38. $\frac{7}{8} - \frac{1}{12}$

39. $\frac{10}{16} - \frac{5}{8}$

40. $\frac{5}{6} - \frac{10}{12}$

41. $\frac{23}{36} - \frac{2}{9}$

42. $\frac{2}{3} - \frac{1}{16}$

43. $\frac{1}{2} + \frac{2}{7} + \frac{3}{14}$

44. $\frac{7}{8} + \frac{5}{6} + \frac{7}{24}$

45. $\frac{5}{30} + \frac{3}{40} + \frac{1}{8}$

46. $\frac{1}{12} + \frac{3}{14} + \frac{4}{21}$

47. $\frac{7}{30} + \frac{2}{5} + \frac{5}{6}$

48. $\frac{1}{12} + \frac{5}{36} + \frac{32}{36}$

Study Example 9 carefully. Then find the value of x in each equation.

49. $x + \frac{1}{7} = \frac{5}{14}$

50. $x + \frac{1}{8} = \frac{7}{16}$

51. $x + \frac{2}{3} = \frac{9}{11}$

52. $x + \frac{3}{4} = \frac{17}{18}$

53. $x - \frac{3}{10} = \frac{4}{15}$

54. $x - \frac{3}{14} = \frac{17}{28}$

Applications

55. ***Cooking*** Rita is baking a cake for a dinner party. The recipe calls for $\frac{2}{3}$ cup sugar for the frosting and $\frac{3}{4}$ cup sugar for the cake. How many total cups of sugar does she need?

56. ***Fitness Training*** Kia is training for a short triathlon. On Monday she swam $\frac{1}{4}$ kilometre and ran $\frac{5}{6}$ kilometre. On Tuesday she swam $\frac{1}{2}$ kilometre and ran $\frac{3}{4}$ kilometre. How many kilometres has she swum so far this week? How many kilometres has she run so far?

57. ***Food Purchase*** Yasmin wants to make a trail mix of nuts and dried fruit. She has $\frac{2}{3}$ kilogram peanuts and $\frac{1}{2}$ kilogram dried cranberries. She purchases $\frac{3}{4}$ kilogram almonds and $\frac{3}{8}$ kilogram raisins to mix with the peanuts and cranberries. After mixing the four ingredients, how many kilograms of nuts and how many kilograms of dried fruit will there be in the trail mix?

58. ***Automobile Maintenance*** Mandy purchased two new steel-belted all-weather radial tires for her car. The tread depth on the new tires measures $\frac{11}{32}$ of an inch. The dealer told her that when the tires have worn down and their tread depth measures $\frac{1}{8}$ of an inch, she should replace the worn tires with new ones. How much will the tread depth decrease over the useful life of the tire?

59. ***Power Outage*** Travis typed $\frac{11}{12}$ of his book report on his computer. Then he printed out $\frac{3}{5}$ of his book report on his computer printer. Suddenly, there was a power outage, and he discovered that he hadn't saved his book report before the power went off. What fractional part of the book report was lost when the power failed?

60. ***Childcare*** An infant's father knows that straight apple juice is too strong for his daughter. Her bottle is $\frac{1}{2}$ full, and he adds $\frac{1}{3}$ of a bottle of water to dilute the apple juice.

(a) How much is there to drink in the bottle after this addition?

(b) If she drinks $\frac{2}{5}$ of the bottle, how much is left?

61. ***Food Purchase*** While he was at the grocery store, Raymond purchased a box of candy for himself. On the way back to the dorm he ate $\frac{1}{4}$ of the candy. As he was putting away the groceries he ate $\frac{1}{2}$ of what was left. There are now six chocolates left in the box. How many chocolates were in the box to begin with?

62. ***Baking*** Peter has $\frac{3}{4}$ cup of cocoa. He needs $\frac{1}{8}$ cup to make brownies, and another $\frac{1}{4}$ cup to make fudge squares. After making the brownies and the fudge, how much cocoa will Peter have left?

63. ***Business Management*** The manager at Fit Factory Health Club was going through his files for 2009 and discovered that only $\frac{7}{10}$ of the members actually used the club. When he checked the numbers from the previous year of 2008, he found that $\frac{7}{8}$ of the members had used the club. What fractional part of the membership represents the decrease in club usage?

Quick Quiz 7 Simplify all answers.

1. Add. $\frac{7}{16} + \frac{3}{4}$

2. Add. $\frac{1}{3} + \frac{5}{7} + \frac{10}{21}$

3. Subtract. $\frac{8}{9} - \frac{7}{15}$

4. **Concept Check** Explain how you would subtract the fractions $\frac{8}{9} - \frac{3}{7}$.

SECTION 8 ADDING AND SUBTRACTING MIXED NUMBERS AND THE ORDER OF OPERATIONS

Student Learning Objectives

After studying this section, you will be able to:

1. **Add mixed numbers.**
2. **Subtract mixed numbers.**
3. **Evaluate fractional expressions using the order of operations.**

1 Adding Mixed Numbers

When adding mixed numbers, it is best to add the fractions together and then add the whole numbers together.

EXAMPLE 1 Add. $3\frac{1}{8} + 2\frac{5}{8}$

Solution

$$\begin{array}{rr} 3 & \frac{1}{8} \\ +2 & \frac{5}{8} \\ \hline 5 & \frac{6}{8} \end{array}$$

Add the whole numbers. $3 + 2 = 5$

Add the fractions. $\frac{1}{8} + \frac{5}{8} = \frac{6}{8}$

$= 5\frac{3}{4}$ ⟵ Reduce $\frac{6}{8} = \frac{3}{4}$

Practice Problem 1 Add. $5\frac{1}{12} + 9\frac{5}{12}$

NOTE TO STUDENT: Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.

If the fraction portions of the mixed numbers do not have a common denominator, we must build the fraction parts to obtain a common denominator before adding.

EXAMPLE 2 Add. $1\frac{2}{7} + 5\frac{1}{3}$

Solution The LCD of $\frac{2}{7}$ and $\frac{1}{3}$ is 21.

$$\frac{2}{7} \times \frac{3}{3} = \frac{6}{21} \qquad \frac{1}{3} \times \frac{7}{7} = \frac{7}{21}$$

Thus $1\frac{2}{7} + 5\frac{1}{3} = 1\frac{6}{21} + 5\frac{7}{21}$.

$$\begin{array}{rcrr} 1\frac{2}{7} & = & 1 & \frac{6}{21} \\ +\,5\frac{1}{3} & = & +\,5 & \frac{7}{21} \\ \hline & & 6 & \frac{13}{21} \end{array}$$

Add the whole numbers. $1 + 5$

Add the fractions. $\frac{6}{21} + \frac{7}{21}$

Practice Problem 2 Add. $6\frac{1}{4} + 2\frac{2}{5}$

If the sum of the fractions is an improper fraction, we convert it to a mixed number and add the whole numbers together.

EXAMPLE 3 Add. $6\frac{5}{6} + 4\frac{3}{8}$

Solution The LCD of $\frac{5}{6}$ and $\frac{3}{8}$ is 24.

$$
\begin{array}{rcl}
6\,\boxed{\frac{5}{6} \times \frac{4}{4}} & = & 6\,\boxed{\frac{20}{24}} \\
+\,4\,\boxed{\frac{3}{8} \times \frac{3}{3}} & = & +\,4\,\boxed{\frac{9}{24}} \\
\hline
 & & 10\,\boxed{\frac{29}{24}}
\end{array}
$$

Add the whole numbers. $\longrightarrow$ 10 $\frac{29}{24}$ $\longleftarrow$ Add the fractions.

$= 10 + \boxed{1\frac{5}{24}}$ Since $\frac{29}{24} = 1\frac{5}{24}$

$= 11\frac{5}{24}$ We add the whole numbers $10 + 1 = 11$.

Practice Problem 3 Add. $7\frac{1}{4} + 3\frac{5}{6}$

2 Subtracting Mixed Numbers

Subtracting mixed numbers is like adding.

EXAMPLE 4 Subtract. $8\frac{5}{7} - 5\frac{5}{14}$

Solution The LCD of $\frac{5}{7}$ and $\frac{5}{14}$ is 14.

$$
\begin{array}{rcl}
8\,\boxed{\frac{5}{7} \times \frac{2}{2}} & = & 8\frac{10}{14} \\
-\,5\frac{5}{14} & = & -5\frac{5}{14} \\
\hline
 & & 3\frac{5}{14}
\end{array}
$$

Subtract the whole numbers. $\longrightarrow 3\frac{5}{14} \longleftarrow$ Subtract the fractions.

Practice Problem 4 Subtract. $12\frac{5}{6} - 7\frac{5}{12}$

Sometimes we must borrow before we can subtract.

EXAMPLE 5 Subtract.

(a) $9\frac{1}{4} - 6\frac{5}{14}$ **(b)** $15 - 9\frac{3}{16}$

Solution This example is fairly challenging. Read through each step carefully. Be sure to have paper and pencil handy and see if you can verify each step.

(a) The LCD of $\frac{1}{4}$ and $\frac{5}{14}$ is 28.

$$\begin{array}{rcl} 9\boxed{\frac{1}{4}\times\frac{7}{7}} & = & 9\frac{7}{28} \\ -\,6\boxed{\frac{5}{14}\times\frac{2}{2}} & = & -\,6\frac{10}{28} \end{array}$$

We cannot subtract $\frac{7}{28} - \frac{10}{28}$, so we will need to borrow.

We borrow 1 from 9 to obtain

$9\frac{7}{28} = 8 + 1\frac{7}{28} = 8 + \frac{35}{28} = 8\frac{35}{28}$

$$\begin{array}{rcl} 9\frac{7}{28} & = & 8\frac{35}{28} \\ -\,6\frac{10}{28} & = & -\,6\frac{10}{28} \\ \hline & & 2\frac{25}{28} \end{array}$$

$\boxed{8 - 6 = 2} \rightarrow 2\frac{25}{28} \leftarrow \boxed{\frac{35}{28} - \frac{10}{28} = \frac{25}{28}}$

(b) The LCD = 16.

$$\begin{array}{rcl} 15 & = & 14\frac{16}{16} \\ -\,9\frac{3}{16} & = & -\,9\frac{3}{16} \\ \hline & & 5\frac{13}{16} \end{array}$$

We borrow 1 from 15 to obtain

$15 = 14 + 1 = 14 + \frac{16}{16} = 14\frac{16}{16}$

$\boxed{14 - 9 = 5} \rightarrow 5\frac{13}{16} \leftarrow \boxed{\frac{16}{16} - \frac{3}{16} = \frac{13}{16}}$

Practice Problem 5 Subtract.

(a) $9\frac{1}{8} - 3\frac{2}{3}$ **(b)** $18 - 6\frac{7}{18}$

***NOTE TO STUDENT:** Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.*

EXAMPLE 6 A plumber had a pipe $5\frac{3}{16}$ inches long for a fitting under the sink. He needed a pipe that was $3\frac{7}{8}$ inches long, so he cut the pipe down. How much of the pipe did he cut off?

Solution We will need to subtract $5\frac{3}{16} - 3\frac{7}{8}$ to find the length that was cut off.

$$\begin{array}{rcl} 5\frac{3}{16} & = & 5\frac{3}{16} \\ -3\frac{7}{8}\times\frac{2}{2} & = & -3\frac{14}{16} \end{array}$$

We borrow 1 from 5 to obtain

$5\frac{3}{16} = 4 + 1\frac{3}{16} = 4 + \frac{19}{16}$

$$\begin{array}{r} 4\frac{19}{16} \\ -\,3\frac{14}{16} \\ \hline 1\frac{5}{16} \end{array}$$

$\boxed{4 - 3 = 1} \rightarrow 1\frac{5}{16} \leftarrow \boxed{\frac{19}{16} - \frac{14}{16} = \frac{5}{16}}$

The plumber had to cut off $1\frac{5}{16}$ inches of pipe.

Practice Problem 6 Hillary and Sam purchased $6\frac{1}{4}$ litres of paint to paint the first floor of their house. They used $4\frac{2}{3}$ litres of paint. How much paint was left over?

ALTERNATIVE METHOD: Add or Subtract Mixed Numbers as Improper Fractions Can mixed numbers be added and subtracted as improper fractions? Yes. Recall Example 5(a).

$$9\frac{1}{4} - 6\frac{5}{14} = 2\frac{25}{28}$$

If we write $9\frac{1}{4} - 6\frac{5}{14}$ using improper fractions, we have $\frac{37}{4} - \frac{89}{14}$. Now we build each of these improper fractions so that they both have the LCD for their denominators.

$$\begin{array}{rcl} \dfrac{37}{4} \boxed{\times \dfrac{7}{7}} & = & \dfrac{259}{28} \\ -\dfrac{89}{14} \boxed{\times \dfrac{2}{2}} & = & -\dfrac{178}{28} \\ \hline & & \dfrac{81}{28} = 2\dfrac{25}{28} \end{array}$$

The same result is obtained as in Example 5(a). This method does not require borrowing. However, you do work with larger numbers. For more practice, see exercises 53–54.

3 Evaluating Fractional Expressions Using the Order of Operations

The order of operations must be considered in problems involving fractions and mixed numbers.

ORDER OF OPERATIONS

With grouping symbols:

Do first	**1.** Perform operations inside parentheses.
↓	**2.** Simplify any expressions with exponents.
	3. Multiply or divide from left to right.
Do last	**4.** Add or subtract from left to right.

EXAMPLE 7 Evaluate. $\frac{3}{4} - \frac{2}{3} \times \frac{1}{8}$

Solution

$$\frac{3}{4} - \frac{2}{3} \times \frac{1}{8} = \frac{3}{4} - \frac{1}{12}$$ First we must multiply $\frac{2}{3} \times \frac{1}{8}$.

$$= \frac{9}{12} - \frac{1}{12}$$ Now we subtract, but first we need to build $\frac{3}{4}$ to an equivalent fraction with a common denominator of 12.

$$= \frac{8}{12}$$ Now we can subtract $\frac{9}{12} - \frac{1}{12}$.

$$= \frac{2}{3}$$ Finally we reduce the fraction.

Practice Problem 7 Evaluate.

$$\frac{3}{5} - \frac{1}{15} \times \frac{10}{13}$$

EXAMPLE 8 Evaluate. $\frac{2}{3} \times \frac{1}{4} + \frac{2}{5} \div \frac{14}{15}$

Solution

$$\frac{2}{3} \times \frac{1}{4} + \frac{2}{5} \div \frac{14}{15} = \frac{1}{6} + \frac{2}{5} \div \frac{14}{15}$$ First we multiply $\frac{2}{3} \times \frac{1}{4}$.

$$= \frac{1}{6} + \frac{2}{5} \times \frac{15}{14}$$ We express the division as a multiplication problem. We invert $\frac{14}{15}$ and multiply.

$$= \frac{1}{6} + \frac{3}{7}$$ Now we perform the multiplication.

$$= \frac{7}{42} + \frac{18}{42}$$ We obtain equivalent fractions with an LCD of 42.

$$= \frac{25}{42}$$ We add the two fractions.

Practice Problem 8 Evaluate.

$$\frac{1}{7} \times \frac{5}{6} + \frac{5}{3} \div \frac{7}{6}$$

***NOTE TO STUDENT:* Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.**

Developing Your Study Skills

Problems with Accuracy

Strive for accuracy. Mistakes are often made because of human error rather than lack of understanding. Such mistakes are frustrating. A simple arithmetic or copying error can lead to an incorrect answer.

These five steps will help you cut down on errors.

1. Work carefully, and take your time. Do not rush through a problem just to get it done.
2. Concentrate on the problem. Sometimes problems become mechanical, and your mind begins to wander. You become careless and make a mistake.
3. Check your problem. Be sure that you copied it correctly from the book.
4. Check your computations from step to step. Check the solution to the problem. Does it work? Does it make sense?
5. Keep practising new skills. Remember the old saying, "Practice makes perfect." An increase in practice results in an increase in accuracy. Many errors are due simply to lack of practice.

There is no magic formula for eliminating all errors, but these five steps will be a tremendous help in reducing them.

Section 8 Exercises

Add or subtract. Express the answer as a mixed number. Simplify all answers.

1. $7\frac{1}{8} + 2\frac{5}{8}$

2. $6\frac{3}{10} + 4\frac{1}{10}$

3. $15\frac{3}{14} - 11\frac{1}{14}$

4. $8\frac{3}{4} - 3\frac{1}{4}$

5. $12\frac{1}{3} + 5\frac{1}{6}$

6. $20\frac{1}{4} + 3\frac{1}{8}$

7. $4\frac{3}{5} + 8\frac{2}{5}$

8. $8\frac{2}{9} + 7\frac{7}{9}$

9. $1 - \frac{3}{7}$

10. $1 - \frac{9}{11}$

11. $1\frac{3}{4} + \frac{5}{16}$

12. $1\frac{2}{3} + \frac{13}{18}$

13. $5\frac{1}{6} + 4\frac{5}{18}$

14. $6\frac{2}{5} + 7\frac{3}{20}$

15. $8\frac{1}{4} - 8\frac{4}{16}$

16. $8\frac{11}{15} - 3\frac{3}{10}$

17. $12\frac{1}{3} - 7\frac{2}{5}$

18. $10\frac{10}{15} - 10\frac{2}{3}$

19. $30 - 15\frac{3}{7}$

20. $25 - 14\frac{2}{11}$

21. $3 + 4\frac{2}{5}$

22. $8 + 2\frac{3}{4}$

23. $14 - 3\frac{7}{10}$

24. $19 - 5\frac{8}{9}$

Add or subtract. Express the answer as a mixed number. Simplify all answers.

25. $\begin{array}{r} 15\frac{4}{15} \\ +26\frac{8}{15} \\ \hline \end{array}$

26. $\begin{array}{r} 22\frac{1}{8} \\ +14\frac{3}{8} \\ \hline \end{array}$

27. $\begin{array}{r} 6\frac{1}{6} \\ +2\frac{1}{4} \\ \hline \end{array}$

28. $\begin{array}{r} 3\frac{2}{3} \\ +4\frac{1}{5} \\ \hline \end{array}$

29. $\begin{array}{r} 3\frac{3}{4} \\ +4\frac{5}{12} \\ \hline \end{array}$

30. $\begin{array}{r} 11\frac{5}{8} \\ +13\frac{1}{2} \\ \hline \end{array}$

31. $\begin{array}{r} 47\frac{3}{10} \\ +26\frac{5}{8} \\ \hline \end{array}$

32. $\begin{array}{r} 34\frac{1}{20} \\ +45\frac{8}{15} \\ \hline \end{array}$

33. $\begin{array}{r} 19\frac{5}{6} \\ -14\frac{1}{3} \\ \hline \end{array}$

34. $\begin{array}{r} 22\frac{7}{9} \\ -16\frac{1}{4} \\ \hline \end{array}$

35. $\begin{array}{r} 6\frac{1}{12} \\ -5\frac{10}{24} \\ \hline \end{array}$

36. $\begin{array}{r} 4\frac{1}{12} \\ -3\frac{7}{18} \\ \hline \end{array}$

37. $\begin{array}{r} 12\frac{3}{20} \\ -\ 7\frac{7}{15} \\ \hline \end{array}$

38. $\begin{array}{r} 8\frac{5}{12} \\ -5\frac{9}{10} \\ \hline \end{array}$

39. $\begin{array}{r} 12 \\ -\ 3\frac{7}{15} \\ \hline \end{array}$

40. $\begin{array}{r} 40 \\ -\ 6\frac{3}{7} \\ \hline \end{array}$

41. $\begin{array}{r} 120 \\ -\ 17\frac{3}{8} \\ \hline \end{array}$

42. $\begin{array}{r} 98 \\ -89\frac{15}{17} \\ \hline \end{array}$

43. $\begin{array}{r} 3\frac{5}{8} \\ 2\frac{2}{3} \\ +7\frac{3}{4} \\ \hline \end{array}$

44. $\begin{array}{r} 4\frac{2}{3} \\ 3\frac{4}{5} \\ +6\frac{3}{4} \\ \hline \end{array}$

Applications

45. ***Mountain Biking*** Lee Hong rode his mountain bike through part of Gatineau Park in Québec. On Wednesday he rode $20\frac{3}{4}$ kilometres. On Thursday he rode $22\frac{3}{8}$ kilometres. What was his total biking distance during those two days?

46. ***Hiking*** Ryan and Omar are planning an afternoon hike. Their map shows three loops measuring $2\frac{1}{8}$ kilometres, $1\frac{5}{6}$ kilometres, and $1\frac{2}{3}$ kilometres. If they hike all three loops, what will their total hiking distance be?

47. ***Bicycling*** Lake Harriet and Lake Calhoun have paved paths around them for runners, walkers, and bicyclists. The distance around Lake Harriet is $2\frac{4}{5}$ kilometres, and the distance around Lake Calhoun is $3\frac{1}{10}$ kilometres. The road connecting the two lakes is $\frac{1}{2}$ kilometre. If Lola rides her bike around both lakes, and uses the connecting road twice, how long is her bike ride?

48. ***Stock Market*** Shanna purchased stock in 1995 at $\$21\frac{3}{8}$ per share. When her son was ready for college, she sold the stock in 2009 at $\$93\frac{5}{8}$ per share. How much did she make per share for her son's tuition?

49. ***Basketball*** Nina and Julie are the two tallest basketball players on their high school team. Nina is $69\frac{3}{4}$ inches tall and Julie is $72\frac{1}{2}$ inches tall. How many inches taller is Julie than Nina?

50. ***Food Purchase*** Julio bought $3\frac{3}{4}$ kilograms of roast turkey and $1\frac{2}{3}$ kilograms of salami at the deli. How many more kilograms of turkey than salami did he buy?

51. ***Food Purchase*** Lara needs 8 kilograms of haddock for her dinner party. At the grocery store, haddock portions weighing $1\frac{3}{4}$ kilograms and $2\frac{1}{6}$ kilograms are placed on the scale.

(a) How many kilograms of haddock are on the scale?

(b) How many more kilograms of haddock does Lara need?

52. ***Medical Care*** A young man has been under a doctor's care to lose weight. His doctor wanted him to lose 46 pounds in the first three months. He lost $17\frac{5}{8}$ pounds the first month and $13\frac{1}{2}$ pounds the second month.

(a) How much did he lose during the first two months?

(b) How much would he need to lose in the third month to reach the goal?

To Think About

Use improper fractions and the Alternative Method as discussed in the text to perform each calculation.

53. $\frac{379}{8} + \frac{89}{5}$

54. $\frac{151}{6} - \frac{130}{7}$

When adding or subtracting mixed numbers, it is wise to estimate your answer by rounding each mixed number to the nearest whole number.

55. Estimate your answer to $35\frac{1}{6} + 24\frac{5}{12}$ by rounding each mixed number to the nearest whole number. Then find the exact answer. How close was your estimate?

56. Estimate your answer to $102\frac{5}{7} - 86\frac{2}{3}$ by rounding each mixed number to the nearest whole number. Then find the exact answer. How close was your estimate?

Evaluate using the correct order of operations.

57. $\frac{6}{7} - \frac{4}{7} \times \frac{1}{3}$

58. $\frac{3}{5} - \frac{1}{3} \times \frac{6}{5}$

59. $\frac{1}{2} + \frac{3}{8} \div \frac{3}{4}$

60. $\frac{3}{4} + \frac{1}{4} \div \frac{5}{3}$

61. $\frac{9}{10} \div \frac{3}{8} \times \frac{5}{8}$

62. $\frac{5}{12} \div \frac{3}{10} \times \frac{9}{5}$

63. $\frac{3}{5} \times \frac{1}{2} + \frac{1}{5} \div \frac{2}{3}$

64. $\frac{5}{6} \times \frac{1}{2} + \frac{2}{3} \div \frac{4}{3}$

65. $\left(\frac{3}{5} - \frac{3}{20}\right) \times \frac{4}{5}$

66. $\left(\frac{1}{3} + \frac{1}{6}\right) \times \frac{5}{11}$

67. $\left(\frac{1}{3}\right)^2 \div \frac{4}{9}$

68. $\left(\frac{1}{4}\right)^2 \div \frac{3}{4}$

69. $\frac{1}{4} \times \left(\frac{2}{3}\right)^2$

70. $\frac{5}{8} \times \left(\frac{2}{5}\right)^2$

71. $\frac{5}{6} \div \left(\frac{2}{3} + \frac{1}{6}\right)^2$

72. $\frac{4}{3} \div \left(\frac{3}{5} - \frac{3}{10}\right)^2$

Quick Quiz 8

1. Add. Express the answer as a mixed number.

$3\frac{4}{5} + 5\frac{3}{8}$

2. Subtract. Express the answer as a mixed number.

$6\frac{5}{12} - 4\frac{7}{10}$

3. Evaluate using the correct order of operations.

$\frac{1}{5} + \frac{3}{10} \div \frac{11}{20}$

4. **Concept Check** Explain how you would evaluate the following expression using the correct order of operations. $\frac{4}{5} - \frac{1}{4} \times \frac{2}{3}$

Section 9 Solving Applied Problems Involving Fractions

1 Solving Real-Life Problems with Fractions

Student Learning Objective

After studying this section, you will be able to:

1. **Solve real-life problems with fractions.**

All problem solving requires the same kind of thinking. In this section we will combine problem-solving skills with our new computational skills with fractions. Sometimes the difficulty is in figuring out what must be done. Sometimes it is in doing the computation. Remember that *estimating* is important in problem solving. We may use the following steps.

1. ***Understand the problem.***
- **(a)** Read the problem carefully.
- **(b)** Draw a picture if this helps you.
- **(c)** Fill in the Mathematics Blueprint.

2. ***Solve.***
- **(a)** Perform the calculations.
- **(b)** State the answer, including the units of measure.

3. ***Check.***
- **(a)** Estimate the answer. Round fractions to the nearest whole number.
- **(b)** Compare the exact answer with the estimate to see if your answer is reasonable.

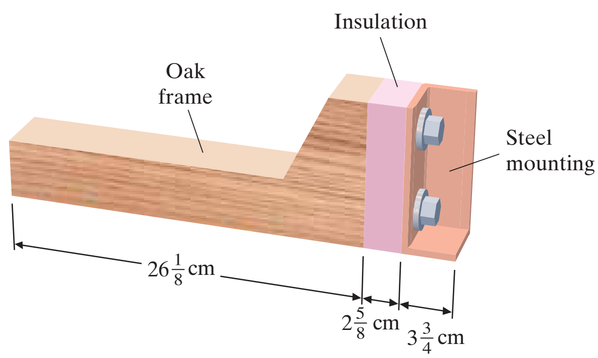
EXAMPLE 1 In designing a modern offshore speedboat, the design engineer has determined that one of the oak frames near the engine housing needs to be $26\frac{1}{8}$ centimetres long. At the end of the oak frame there will be $2\frac{5}{8}$ centimetres of insulation. Finally, there will be a steel mounting that is $3\frac{3}{4}$ centimetres long. When all three items are assembled, how long will the oak frame and insulation and steel mounting extend?

Solution

1. ***Understand the problem.***

We draw a picture to help us.

Then we fill in the Mathematics Blueprint.



Mathematics Blueprint for Problem Solving

Gather the Facts	What Am I Asked to Do?	How Do I Proceed?	Key Points to Remember
Oak frame: $26\frac{1}{8}$ cm Insulation: $2\frac{5}{8}$ cm Steel mounting: $3\frac{3}{4}$ cm	Find the total length.	Add the lengths of the three items.	When adding mixed numbers, add the whole numbers first and then add the fractions.

2. *Solve and state the answer.*

Add the three amounts. $26\frac{1}{8} + 2\frac{5}{8} + 3\frac{3}{4}$

$$\begin{aligned} \text{LCD} = 8 \quad & 26\frac{1}{8} && = \quad 26\frac{1}{8} \\ & 2\frac{5}{8} && = \quad 2\frac{5}{8} \\ & +3\boxed{\frac{3}{4} \times \frac{2}{2}} && = +\ 3\frac{6}{8} \\ \hline & && \quad 31\frac{12}{8} = 32\frac{4}{8} = 32\frac{1}{2} \end{aligned}$$

The entire assembly will be $32\frac{1}{2}$ centimetres.

3. *Check.* Estimate the sum by rounding each fraction to the nearest whole number.

Thus $\quad 26\frac{1}{8} + 2\frac{5}{8} + 3\frac{3}{4}$

becomes $\quad 26 + 3 + 4 = 33$

This is close to our answer, $32\frac{1}{2}$. Our answer seems reasonable.

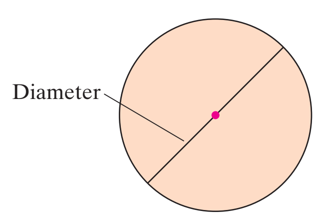
One of the most important uses of estimation in mathematics is in the calculation of problems involving fractions. People find it easier to detect significant errors when working with whole numbers. However, the extra steps involved in the calculations with fractions and mixed numbers often distract our attention from an error that we should have detected.

Thus it is particularly critical to take the time to check your answer by estimating the results of the calculation with whole numbers. Be sure to ask yourself, is this answer reasonable? Does this answer seem realistic? Only by estimating our results with whole numbers will we be able to answer that question. It is this estimating skill that you will find more useful in your own life as a consumer and as a citizen.

***NOTE TO STUDENT:** Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.*

Practice Problem 1 Nicole required the following amounts of gas for her farm tractor in the last three fill-ups: $18\frac{7}{10}$ litres, $15\frac{2}{5}$ litres, and $14\frac{1}{2}$ litres. How many litres did she need altogether?

The word *diameter* has two common meanings. First, it means a line segment that passes through the centre of and intersects a circle twice. It has its endpoints on the circle. Second, it means the *length* of this segment.



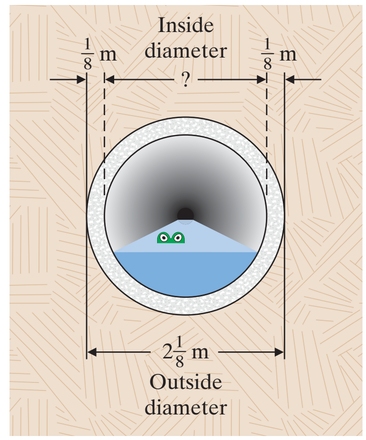
EXAMPLE 2 What is the inside diameter of (distance across) a cement storm drain pipe that has an outside diameter of $2\frac{1}{8}$ metres and is $\frac{1}{8}$ metre thick?

Solution

1. ***Understand the problem.*** Read the problem carefully. Draw a picture. The picture is in the margin on the right. Now fill in the Mathematics Blueprint.

Mathematics Blueprint for Problem Solving

Gather the Facts	What Am I Asked to Do?	How Do I Proceed?	Key Points to Remember
Outside diameter is $2\frac{1}{8}$ metres. Thickness is $\frac{1}{8}$ metre on both ends of the diameter.	Find the *inside* diameter of the pipe.	Add the two measures of thickness. Then subtract this total from the outside diameter.	Since the LCD $= 8$, all fractions must have this denominator.



2. ***Solve and state the answer.*** Add the two thickness measurements together. Adding $\frac{1}{8} + \frac{1}{8} = \frac{2}{8}$ gives the total thickness of the pipe, $\frac{2}{8}$ metre. We will not reduce $\frac{2}{8}$ since the LCD is 8.

We subtract the total of the two thickness measurements from the outside diameter.

$$\begin{array}{rcr} 2\frac{1}{8} & = & 1\frac{9}{8} \\ -\ \frac{2}{8} & = & -\ \frac{2}{8} \\ \hline & & 1\frac{7}{8} \end{array}$$

We borrow 1 from 2 to get $1 + 1\frac{1}{8}$ or $1\frac{9}{8}$.

The inside diameter is $1\frac{7}{8}$ metres.

3. ***Check.*** We will work backward to check. We will use the exact values. If we have done our work correctly, $\frac{1}{8}$ metre $+ 1\frac{7}{8}$ metres $+ \frac{1}{8}$ metre should add up to the outside diameter, $2\frac{1}{8}$ metres.

$$\frac{1}{8} + 1\frac{7}{8} + \frac{1}{8} \stackrel{?}{=} 2\frac{1}{8}$$

$$1\frac{9}{8} \stackrel{?}{=} 2\frac{1}{8}$$

$$2\frac{1}{8} = 2\frac{1}{8} \quad \checkmark$$

Our answer of $1\frac{7}{8}$ metres is correct.

Practice Problem 2 A postcard is $12\frac{1}{4}$ centimetres long. We want a $1\frac{3}{8}$-centimetre border on the top and a 2-centimetre border on the bottom. What is the length of the inside portion of the postcard?

***NOTE TO STUDENT:** Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.*

EXAMPLE 3 On Tuesday, Michael earned $\$8\frac{1}{4}$ per hour working for eight hours. He also earned overtime pay, which is $1\frac{1}{2}$ times his regular rate of $\$8\frac{1}{4}$, for four hours on Tuesday. How much pay did he earn altogether on Tuesday?

Solution

1. ***Understand the problem.*** We draw a picture of the parts of Michael's pay on Tuesday.
   Michael's earnings on Tuesday are the sum of two parts:

Pay at regular pay rate	+	Pay at overtime pay rate	=	Total pay for the day

   Now fill in the Mathematics Blueprint.

Mathematics Blueprint for Problem Solving

Gather the Facts	What Am I Asked to Do?	How Do I Proceed?	Key Points to Remember
He works eight hours at $\$8\frac{1}{4}$ per hour. He works four hours at the overtime rate, $1\frac{1}{2}$ times the regular rate.	Find his total pay for Tuesday.	Find out how much he is paid for regular time. Find out how much he is paid for overtime. Then add the two.	The overtime rate is $1\frac{1}{2}$ multiplied by the regular rate.

2. ***Solve and state the answer.*** Find his overtime pay rate.

$$1\frac{1}{2} \times 8\frac{1}{4} = \frac{3}{2} \times \frac{33}{4} = \frac{\$99}{8} \text{ per hour}$$

   We leave our answer as an improper fraction because we will need to multiply it by another fraction.
   How much was he paid for regular time? For overtime?

   For eight regular hours, he earned $8 \times 8\frac{1}{4} = \overset{2}{\cancel{8}} \times \frac{33}{\underset{1}{\cancel{4}}} = \$66.$

   For four overtime hours, he earned $\overset{1}{\cancel{4}} \times \frac{99}{\underset{2}{\cancel{8}}} = \frac{99}{2} = \$49\frac{1}{2}.$

   Now we add to find the total pay.

$$\begin{array}{rl} \$66 & \text{Pay at regular pay rate} \\ +\$49\frac{1}{2} & \text{Pay at overtime pay rate} \\ \hline \end{array}$$

   Michael earned $\$115\frac{1}{2}$ working on Tuesday. This is the same as \$115.50.

3. *Check.* We estimate his regular pay rate at \$8 per hour.

We estimate his overtime pay rate at $1\frac{1}{2} \times 8 = \frac{3}{2} \times 8 = 12$ or \$12 per hour.

$$8 \text{ hours} \times \$8 \text{ per hour} = \$64 \text{ regular pay}$$
$$4 \text{ hours} \times \$12 \text{ per hour} = \$48 \text{ overtime pay}$$

Estimated sum. $\$64 + \$48 \approx \$60 + \$50 = \$110$

\$110 is close to our calculated value, $\$115\frac{1}{2}$, so our answer is reasonable. ✓

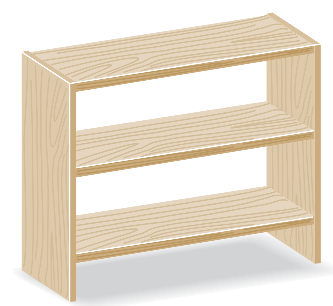
Practice Problem 3 A tent manufacturer uses $8\frac{1}{4}$ metres of waterproof duck cloth to make a regular tent. She uses $1\frac{1}{2}$ times that amount to make a large tent. How many metres of cloth will she need to make 6 regular tents and 16 large tents?

EXAMPLE 4 Alicia is buying some 8-foot boards for shelving. She wishes to make two bookcases, each with three shelves. Each shelf will be $3\frac{1}{4}$ feet long.

(a) How many boards does she need to buy?

(b) How many linear feet of shelving are actually needed to build the bookcases?

(c) How many linear feet of shelving will be left over?

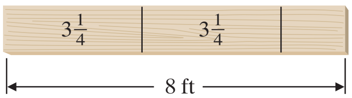


Solution

1. *Understand the problem.* Draw a sketch of a bookcase. Each bookcase will have three shelves. Alicia is making two such bookcases. (Alicia's boards are for the shelves, not the sides.)
Now fill in the Mathematics Blueprint.

Mathematics Blueprint for Problem Solving

Gather the Facts	What Am I Asked to Do?	How Do I Proceed?	Key Points to Remember
She needs three shelves for each bookcase. Each shelf is $3\frac{1}{4}$ feet long. She will make two bookcases. Shelves are cut from 8-foot boards.	Find out how many boards to buy. Find out how many feet of board are needed for shelves and how many feet will be left over.	First find out how many $3\frac{1}{4}$-foot shelves she can get from one board. Then see how many boards she needs to make all six shelves.	Each time she cuts up an 8-foot board, she will get some shelves and some leftover wood.



2. ***Solve and state the answer.*** We want to know how many $3\frac{1}{4}$-foot boards are in an 8-foot board. By drawing a rough sketch, we would probably guess the answer is 2. To find exactly how many $3\frac{1}{4}$-foot-long pieces are in 8 feet, we will use division.

$$8 \div 3\frac{1}{4} = \frac{8}{1} \div \frac{13}{4} = \frac{8}{1} \times \frac{4}{13} = \frac{32}{13} = 2\frac{6}{13}$$

She will get two shelves from each board, and some wood will be left over.

(a) How many boards does Alicia need to build two bookcases? For two bookcases, she needs six shelves. She will get two shelves out of each board. $6 \div 2 = 3$. She will need three 8-foot boards.

(b) How many linear feet of shelving are actually needed to build the bookcases?
She needs 6 shelves at $3\frac{1}{4}$ feet.

$$6 \times 3\frac{1}{4} = \overset{3}{\cancel{6}} \times \frac{13}{\underset{2}{\cancel{4}}} = \frac{39}{2} = 19\frac{1}{2}$$

A total of $19\frac{1}{2}$ linear feet of shelving is needed.

(c) How many linear feet of shelving will be left over?
Each time she uses one board she will have

$$8 - 3\frac{1}{4} - 3\frac{1}{4} = 8 - \left(3\frac{1}{4} + 3\frac{1}{4}\right) = 8 - 6\frac{1}{2} = 1\frac{1}{2}$$

feet left over. Each of the three boards will have $1\frac{1}{2}$ feet left over.

$$3 \times 1\frac{1}{2} = 3 \times \frac{3}{2} = \frac{9}{2} = 4\frac{1}{2}$$

A total of $4\frac{1}{2}$ linear feet of shelving will be left over.

3. ***Check.*** Work backward. See if you can check that with three 8-foot boards you

(a) can make the six shelves for the two bookcases.
(b) will use exactly $19\frac{1}{2}$ linear feet to make the shelves.
(c) will have exactly $4\frac{1}{2}$ linear feet left over.

The check is left to you.

Practice Problem 4 Michael is purchasing 4-metre boards for shelving. He wishes to make two bookcases, each with four shelves. Each shelf will be $1\frac{1}{4}$ metres long.

(a) How many boards does he need to buy?
(b) How many linear metres of shelving are actually needed to build the bookcases?
(c) How many linear metres of shelving will be left over?

Another useful method for solving applied problems is called "Do a similar, simpler problem." When a problem seems difficult to understand because of the fractions, change the problem to an easier but similar problem. Then decide how to solve the simpler problem and use the same steps to solve the original problem. For example:

> How many litres of water can a tank hold if its volume is $58\frac{2}{3}$ cubic metres? (1 cubic metre holds about 1000 litres.)

A similar, easier problem would be: "If 1 cubic metre holds 1000 litres and a tank holds 60 cubic metres, how many litres of water does the tank hold?"

The easier problem can be read more quickly and seems to make more sense. Probably we will see how to solve the easier problem right away: "I can find the number of litres by multiplying 1000×60." Therefore we can solve the first problem by multiplying $1000 \times 58\frac{2}{3}$ to obtain the number of litres of water. See the next example.

EXAMPLE 5 A fishing boat travelled $69\frac{3}{8}$ nautical miles in $3\frac{3}{4}$ hours. How many knots (nautical miles per hour) did the fishing boat average?

Solution

1. ***Understand the problem.*** Let us think of a simpler problem. If a boat travelled 70 nautical miles in 4 hours, how many knots did it average? We would divide distance by time.

$$70 \div 4 = \text{average speed}$$

Likewise in our original problem we need to divide distance by time.

$$69\frac{3}{8} \div 3\frac{3}{4} = \text{average speed}$$

Now fill in the Mathematics Blueprint.

Mathematics Blueprint for Problem Solving

Gather the Facts	What Am I Asked to Do?	How Do I Proceed?	Key Points to Remember
Distance is $69\frac{3}{8}$ nautical miles. Time is $3\frac{3}{4}$ hours.	Find the average speed of the boat.	Divide the distance in nautical miles by the time in hours.	You must change the mixed numbers to improper fractions before dividing.

2. ***Solve and state the answer.*** Divide distance by time to get speed in nautical miles per hour or knots.

$$69\frac{3}{8} \div 3\frac{3}{4} = \frac{555}{8} \div \frac{15}{4} = \frac{\overset{37}{\cancel{555}}}{\underset{2}{\cancel{8}}} \cdot \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{15}}}$$

$$= \frac{37}{2} \cdot \frac{1}{1} = \frac{37}{2} = 18\frac{1}{2}$$

The speed of the boat was $18\frac{1}{2}$ knots.

Bill Stanton/ImageState Media Partners Limited

3. *Check.*

We estimate $69\frac{3}{8} \div 3\frac{3}{4}$.

$$\text{Use } 70 \div 4 = 17\frac{1}{2} \text{ knots}$$

Our estimate is close to the calculated value.

Our answer is reasonable. ✓

NOTE TO STUDENT: Fully worked-out solutions to all of the Practice Problems can be found at the end of the module.

Practice Problem 5 Alfonso travelled $199\frac{3}{4}$ kilometres in his car and used $8\frac{1}{2}$ litres of gas. How many kilometres per litre did he get?

Be sure to allow extra time to read over Examples 1–5 and Practice Problems 1–5. Many students find it is helpful to study them on two different days. This allows you additional time to really understand the steps of reasoning involved.

Developing Your Study Skills

Why Study Mathematics?

Students often question the value of mathematics. They see little real use for it in their everyday lives. However, mathematics is often the key that opens the door to a better-paying job.

In our present-day technological world, many people use mathematics daily. Many vocational and professional areas—such as the fields of business, statistics, economics, psychology, finance, computer science, chemistry, physics, engineering, electronics, nuclear energy, banking, quality control, and teaching—require a certain level of expertise in mathematics. Those who want to work in these fields must be able to function at a given mathematical level. Those who cannot will not be able to enter these job areas.

So, whatever your field, be sure to realize the importance of mastering the basics of this course. It is very likely to help you advance to the career of your choice.

SECTION 9 EXERCISES

You may benefit from using the Mathematics Blueprint for Problem Solving when solving the following exercises.

Applications

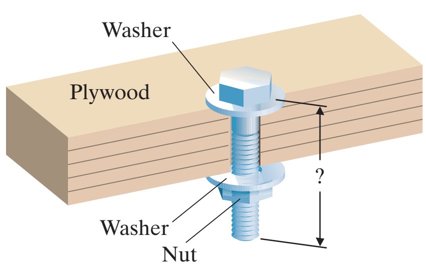
▲**1.** ***Geometry*** A triangle has three sides that measure $8\frac{1}{3}$ cm, $5\frac{4}{5}$ cm, and $9\frac{3}{10}$ cm. What is the perimeter of (total distance around) the triangle?

2. ***Automobile Travel*** On Tuesday, Sally drove $10\frac{1}{2}$ kilometres while running errands. On Friday and Saturday, she had more errands to run and drove $6\frac{1}{3}$ kilometres and $12\frac{1}{4}$ kilometres, respectively. How many total kilometres did Sally drive this week while running errands?

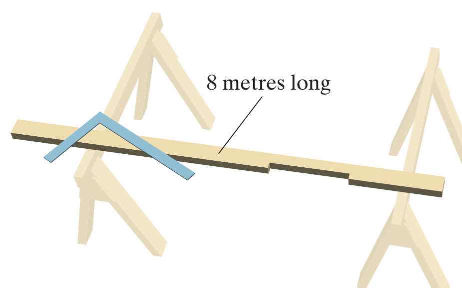
3. ***Wildlife*** In 2006, only 700 mountain gorillas remained in the world. Of these, about $\frac{5}{9}$ of them were living in a mountain range along the borders of Congo, Rwanda, and Uganda. How many gorillas were living in this mountain range? Round your answer to the nearest whole number.

4. ***Consumer Decisions*** Between 2005 and 2006, prices on many electronic devices went down. The average price of a flat-panel television in 2005 was \$1190. In 2006, the average price was about $\frac{4}{5}$ as much. What was the average price of a flat-panel television in 2006?

5. ***Carpentry*** A bolt extends through $\frac{3}{4}$-inch-thick plywood, two washers that are each $\frac{1}{16}$ inch thick, and a nut that is $\frac{3}{16}$ inch thick. The main body of the bolt must be $\frac{1}{2}$ inch longer than the sum of the thicknesses of plywood, washers, and nut. What is the minimum length of the bolt?



6. ***Carpentry*** A carpenter is using an 8-metre length of wood for a frame. The carpenter needs to cut a notch in the wood that is $4\frac{7}{8}$ metres from one end and $1\frac{2}{3}$ metres from the other end. How long does the notch need to be?



7. ***Running a Marathon*** Hank is running the Terry Fox Run, which is $26\frac{1}{5}$ kilometres long. At $6\frac{3}{4}$ kilometres from the start, he meets his wife, who is cheering him on. $9\frac{1}{2}$ kilometres farther down the course, he sees some friends from his running club volunteering at a water stop. Once he passes his friends, how many more kilometres does Hank have left to run?

8. ***Carpentry*** Norman Olerud makes birdhouses as a hobby. He has a long piece of lumber that measures $14\frac{1}{4}$ feet. He needs to cut it into pieces that are $\frac{3}{4}$ foot long for the birdhouse floors. How many floors will he be able to cut from the long piece?

9. ***Personal Finance*** Javier earned $\$10\frac{1}{2}$ per hour for 8 hours of work on Saturday. His manager asked him to stay for an additional 4 hours, for which he was paid $1\frac{1}{2}$ times the regular rate. How much did Javier earn on Saturday?

10. ***Food Purchase*** For a party of the British Literature Club using all "English foods," Nancy bought a $10\frac{2}{3}$-kilogram wheel of Stilton cheese, to go with the pears and the apples, at $\$8\frac{3}{4}$ per kilogram. How much did the wheel of Stilton cheese cost?

▲ **11.** ***Geometry*** How many litres can a tank hold that has a volume of $36\frac{3}{4}$ cubic metres? (Assume that 1 cubic metre holds 1000 litres.)

▲ **12.** ***Geometry*** A tank can hold a volume of $7\frac{1}{4}$ cubic metres. If it is filled with water, what is the mass of the water? (Assume that 1 cubic metre of water has a mass of 1000 kilograms.)

13. ***Titanic Disaster*** The night of the *Titanic* cruise ship disaster, the captain decided to run his ship at $22\frac{1}{2}$ knots (nautical miles per hour). The *Titanic* travelled at that speed for $4\frac{3}{4}$ hours before it met its tragic demise. How far did the *Titanic* travel at this excessive speed before the disaster?

14. ***Personal Finance*** William built a porch for his neighbour and got paid \$1200. He gave $\frac{1}{10}$ of this to his brother to pay back a debt. He used $\frac{1}{3}$ of it to pay bills and used $\frac{1}{6}$ to pay his helper. How much of the \$1200 did William have left?

15. ***Personal Finance*** Noriko earns \$660 per week. She has $\frac{1}{5}$ of her income deducted for federal taxes, $\frac{1}{15}$ of her income deducted for provincial taxes, and $\frac{1}{20}$ of her income deducted for medical coverage. How much per week is left for Noriko after these three deductions?

16. ***Real Estate*** Dan and Estella are saving for a down payment on a house. Their total take-home pay is \$960 per week. They have allotted $\frac{1}{4}$ of their weekly income for rent, $\frac{1}{10}$ for car insurance, and $\frac{1}{3}$ for all other expenses including groceries, clothing, entertainment, and monthly bills. How much is left per week to be saved for their down payment?

17. ***Making Jewellery*** Emily makes bracelets and sells them for $\$9\frac{1}{2}$. She has a long piece of wire that measures 20 metres. Each bracelet requires $\frac{3}{5}$ metre to make.

(a) How many bracelets can Emily make from the long piece of wire?

(b) How much wire is left over?

(c) If Emily sells all the bracelets, how much money will she make?

▲ **18.** ***Home Improvement*** The Costellos are having new carpet and moulding installed in their sunroom. The room measures $2\frac{1}{2}$ metres by $3\frac{2}{3}$ metres.

(a) If new carpet costs \$9 per square metre to install, how much will the new carpet cost?

(b) The new moulding will be placed around the room where the wall and ceiling meet. How many metres of moulding will they need?

19. ***Food Purchase*** Cecilia bought a loaf of sourdough bread that was made by a local gourmet bakery. The label said that the bread, plus its fancy box, weighed 1.85 kg, which is equivalent to $18\frac{1}{2}$ hg, in total. Of this, $1\frac{1}{4}$ hg turned out to be the weight of the ribbon. The box weighed $3\frac{1}{8}$ hg.

(a) How many hectograms of bread did she actually buy?

(b) The box stated its net weight as equivalent to $14\frac{3}{4}$ hg. (This means that she should have found $14\frac{3}{4}$ hg of gourmet sourdough bread in the box.) How much in error was this measurement?

20. ***Cooking*** Marnie has $12\frac{1}{2}$ cups of flour. She wants to make two pies, each requiring $1\frac{1}{4}$ cups of flour, and three cakes, each requiring $2\frac{1}{8}$ cups. How much flour will be left after Marnie makes the pies and cakes?

21. ***Coast Guard Boat Operation*** The largest Coast Guard boat stationed at Vancouver can travel $160\frac{1}{8}$ nautical miles in $5\frac{1}{4}$ hours.

(a) At how many knots is the boat travelling?

(b) At this speed, how long would it take the Coast Guard boat to travel $213\frac{1}{2}$ nautical miles?

22. ***Water Ski Boat*** Russ and Norma's Mariah water ski boat can travel $72\frac{7}{8}$ nautical miles in $2\frac{3}{4}$ hours.

(a) At how many knots is the boat travelling?

(b) At this speed, how long would it take their water ski boat to travel $92\frac{3}{4}$ nautical miles?

▲ **23.** ***Farming*** A Saskatchewan wheat farmer has a storage bin with a capacity of $6856\frac{1}{4}$ cubic metres.

(a) If a bushel of wheat is $\frac{1}{30}$ cubic metre, how many bushels can the storage bin hold?

(b) If a farmer wants to make a new storage bin $1\frac{3}{4}$ times larger, how many cubic metres will it hold?

(c) How many bushels will the new bin hold?

▲ **24.** ***Farming*** A wheat farmer from Manitoba has a storage bin with a capacity of $8693\frac{1}{3}$ cubic metres.

(a) If a bushel of wheat is $\frac{1}{25}$ cubic metre, how many bushels can the storage bin hold?

(b) If a farmer wants to make a new storage bin $1\frac{1}{3}$ times larger, how many cubic metres will it hold?

(c) How many bushels will the new bin hold?

Quick Quiz 9

1. Marcia wants to put wall-to-wall carpet in her rec-room. The room measures $15\frac{3}{4}$ metres by $10\frac{2}{3}$ metres. How many square metres of carpeting does she need?

2. Ken Thompson shipped out $41\frac{3}{5}$ kilograms of electrical supplies. The supplies are placed in individual packets that weigh $2\frac{3}{5}$ kilograms each. How many packets did he ship out?

3. Lexi bicycled $1\frac{1}{8}$ kilometres from Spruce St. to Fogg St. She then travelled $1\frac{1}{2}$ kilometres from Fogg St. to Westmount Blvd. Finally she travelled $2\frac{3}{4}$ kilometres from Westmount Blvd. to Spruce St. How far did she travel on her bicycle? Express your answer as a mixed number.

4. **Concept Check** A trail to a peak on Crowsnest Mountain is $3\frac{3}{5}$ kilometres long. Caleb started hiking on the trail and stopped after walking $1\frac{7}{8}$ kilometres to take a break. Explain how you would find how far he still has to go to get to the peak.

Putting Your Skills to Work: Use Math to Save Money

FINDING EXTRA MONEY EACH MONTH

Do you find yourself running short of money each month? Do you wish you could find a little extra cash for yourself? Is there some daily habit that costs money that maybe you could give up? Let's start with smoking cigarettes. (If you don't smoke, think of some other example, perhaps your daily cup of coffee, where you spend money.) Now consider the story of a young couple, Tricia and Jack.

Lim ChewHow/Shutterstock

Tricia and Jack both used to smoke cigarettes. Then Tricia experienced some health problems and had to go to the hospital for several days. Tricia and Jack both decided they were done with smoking. It was pretty hard to quit, and at first they just focused on how hard it was for them. But then Tricia and Jack noticed they were having more money left over at the end of the month.

They got to thinking. Where they live, cigarettes cost $6 a pack, and they were both pack-a-day smokers. How much had they spent together per month on cigarettes? (Figure 30 days for an average month.)

1. (a) Find out how much Tricia and Jack spent in a month (30 days) on cigarettes.

(b) Use your answer from (a) to find out how much they spent in 12 months on cigarettes.

Tricia and Jack enjoyed smoking but knew it was bad for their health. So they decided to put the money they saved by not smoking into a savings account for something they would really enjoy. They thought they could purchase a really nice plasma television for $2000. If they put the money they saved each month into the savings account, would there be enough money by Tricia's birthday (which is 7 months from now)?

2. (a) Find out if they would save enough money in 7 months for a television.

(b) Would there be extra money for a birthday celebration dinner for Tricia? If so, how much?

3. If Tricia and Jack found a plasma television on sale that only costs $\frac{3}{4}$ of what the television costs in problem 2 (above), how much money would be available for the birthday dinner?

Some cities and provinces across Canada are imposing taxes on the sale of cigarettes as a way to offset the cost of health care for people who suffer from smoking-related medical issues. These taxes make smoking cigarettes even more costly. For example, in July of 2008 in Toronto, a pack of cigarettes cost approximately $10.

If Tricia and Jack lived in Toronto and each smoked a pack of cigarettes per day, how much money would they spend on cigarettes for a month?

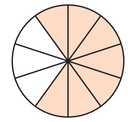
4. (a) Find out how much Tricia and Jack would spend in 30 days on cigarettes in Toronto.

(b) Use your answer from (a) to find out how much Tricia and Jack would spend on cigarettes in 12 months in Toronto.

(c) How much more is this amount than the amount you found in problem 1(b) above?

5. Can you think of one extra expense you could eliminate so you could save money to purchase a big ticket item? Calculate the savings.

Module Organizer

Topic	Procedure	Examples
Concept of a fractional part.	The numerator is the number of parts selected. The denominator is the number of total parts.	What part of this sketch is shaded?  $\frac{7}{10}$
Prime factorization.	Prime factorization is the writing of a number as the product of prime numbers.	Write the prime factorization of 36. $36 = 4 \times 9$; $4 = 2 \times 2$, $9 = 3 \times 3$; $= 2 \times 2 \times 3 \times 3$
Reducing fractions.	**1.** Factor numerator and denominator into prime factors. **2.** Divide out factors common to numerator and denominator.	Reduce. $\frac{54}{90}$ $\frac{54}{90} = \frac{\overset{1}{\cancel{2}} \times \overset{1}{\cancel{3}} \times \overset{1}{\cancel{3}} \times 3}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{3}} \times 5} = \frac{3}{5}$
Changing a mixed number to an improper fraction.	**1.** Multiply whole number by denominator. **2.** Add product to numerator. **3.** Place sum over denominator.	Write as an improper fraction. $7\frac{3}{4} = \frac{7 \times 4 + 3}{4} = \frac{28 + 3}{4} = \frac{31}{4}$
Changing an improper fraction to a mixed number.	**1.** Divide denominator into numerator. **2.** The quotient is the whole number. **3.** The fraction is the remainder over the divisor.	Change to a mixed number. $\frac{32}{5}$ $5\overline{)32} = 6$ remainder 2 (30, 2): $\frac{32}{5} = 6\frac{2}{5}$
Multiplying fractions.	**1.** Divide out common factors from the numerators and denominators whenever possible. **2.** Multiply numerators. **3.** Multiply denominators.	Multiply. $\frac{3}{7} \times \frac{5}{13} = \frac{15}{91}$ Multiply. $\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{8}}} \times \frac{\overset{2}{\cancel{16}}}{\underset{3}{\cancel{15}}} = \frac{2}{3}$
Multiplying mixed and/or whole numbers.	**1.** Change any whole numbers to fractions with a denominator of 1. **2.** Change any mixed numbers to improper fractions. **3.** Use multiplication rule for fractions.	Multiply. $7 \times 3\frac{1}{4}$ $\frac{7}{1} \times \frac{13}{4} = \frac{91}{4}$ or $22\frac{3}{4}$
Dividing fractions.	To divide two fractions, we invert the second fraction and multiply.	Divide. $\frac{3}{7} \div \frac{2}{9} = \frac{3}{7} \times \frac{9}{2} = \frac{27}{14}$ or $1\frac{13}{14}$
Dividing mixed numbers and/or whole numbers.	**1.** Change any whole numbers to fractions with a denominator of 1. **2.** Change any mixed numbers to improper fractions. **3.** Use rule for division of fractions.	Divide. $8\frac{1}{3} \div 5\frac{5}{9} = \frac{25}{3} \div \frac{50}{9}$ $= \frac{\overset{1}{\cancel{25}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{50}}} = \frac{3}{2}$ or $1\frac{1}{2}$
Finding the least common denominator.	**1.** Write each denominator as the product of prime factors. **2.** List all the prime factors that appear in both products. **3.** Form a product of those factors, using each factor the greatest number of times it appears in any denominator.	Find LCD of $\frac{1}{10}, \frac{3}{8}$, and $\frac{7}{25}$. $10 = 2 \times 5$ $8 = 2 \times 2 \times 2$ $25 = 5 \times 5$ LCD $= 2 \times 2 \times 2 \times 5 \times 5 = 200$

(*Continued on next page*)

Topic	Procedure	Examples
Building fractions.	**1.** Find how many times the original denominator can be divided into the new denominator. **2.** Multiply that value by numerator and denominator of original fraction.	Build $\frac{5}{7}$ to an equivalent fraction with a denominator of 42. First we find $7\overline{)42}$ = 6. Then we multiply the numerator and denominator by 6. $\frac{5}{7} \times \frac{6}{6} = \frac{30}{42}$
Adding or subtracting fractions with a common denominator.	**1.** Add or subtract the numerators. **2.** Keep the common denominator.	Add. $\frac{3}{13} + \frac{5}{13} = \frac{8}{13}$ Subtract. $\frac{15}{17} - \frac{12}{17} = \frac{3}{17}$
Adding or subtracting fractions without a common denominator.	**1.** Find the LCD of the fractions. **2.** Build each fraction, if needed, to obtain the LCD in the denominator. **3.** Follow the steps for adding and subtracting fractions with the same denominator.	Add. $\frac{1}{4} + \frac{3}{7} + \frac{5}{8}$ LCD = 56 $\frac{1}{4} \times \frac{14}{14} + \frac{3}{7} \times \frac{8}{8} + \frac{5}{8} \times \frac{7}{7}$ $= \frac{14}{56} + \frac{24}{56} + \frac{35}{56} = \frac{73}{56}$ or $1\frac{17}{56}$
Adding mixed numbers.	**1.** Change fractional parts to equivalent fractions with LCD as a denominator, if needed. **2.** Add whole numbers and fractions separately. **3.** If improper fractions occur, change to mixed numbers and simplify.	Add. $6\frac{3}{4} + 2\frac{5}{8}$ $6\boxed{\frac{3}{4} \times \frac{2}{2}} = 6\frac{6}{8}$ $+\,2\frac{5}{8} \quad = +\,2\frac{5}{8}$ $8\frac{11}{8} = 9\frac{3}{8}$
Subtracting mixed numbers.	**1.** Change fractional parts to equivalent fractions with LCD as a denominator, if needed. **2.** If necessary, borrow from whole number to subtract fractions. **3.** Subtract whole numbers and fractions separately.	Subtract. $8\frac{1}{5} - 4\frac{2}{3}$ $8\boxed{\frac{1}{5} \times \frac{3}{3}} = 8\frac{3}{15} = 7\frac{18}{15}$ $-\,4\boxed{\frac{2}{3} \times \frac{5}{5}} = -\,4\frac{10}{15} = -\,4\frac{10}{15}$ $3\frac{8}{15}$
Order of operations.	With grouping symbols: Do first **1.** Perform operations inside parentheses. ↓ **2.** Simplify any expressions with exponents. **3.** Multiply or divide from left to right. Do last **4.** Add or subtract from left to right.	$\frac{5}{6} \div \left(\frac{4}{5} - \frac{7}{15}\right)$ First combine numbers inside the parentheses. $\frac{5}{6} \div \left(\frac{12}{15} - \frac{7}{15}\right)$ Transform $\frac{4}{5}$ to equivalent fraction $\frac{12}{15}$. $\frac{5}{6} \div \frac{1}{3}$ Subtract the two fractions inside the parentheses and reduce. $\frac{5}{6} \times \frac{3}{1}$ Invert the second fraction and multiply. $\frac{5}{2}$ or $2\frac{1}{2}$ Simplify.

Procedure for Solving Applied Problems

Using the Mathematics Blueprint for Problem Solving

In solving an applied problem with fractions, students may find it helpful to complete the following steps. You will not use all the steps all of the time. Choose the steps that best fit the conditions of the problem.

1. Understand the problem.

- **(a)** Read the problem carefully.
- **(b)** Draw a picture if this helps you to visualize the situation. Think about what facts you are given and what you are asked to find.
- **(c)** It may help to write a similar, simpler problem to get started and to determine what operation to use.
- **(d)** Use the Mathematics Blueprint for Problem Solving to organize your work. Follow these four parts.
  - **1.** Gather the facts. (Write down specific values given in the problem.)
  - **2.** What am I asked to do? (Identify what you must obtain for an answer.)
  - **3.** How do I proceed? (Decide what calculations need to be done.)
  - **4.** Key points to remember. (Record any facts, warnings, formulas, or concepts you think will be important as you solve the problem.)

2. Solve and state the answer.

- **(a)** Perform the necessary calculations.
- **(b)** State the answer, including the unit of measure.

3. Check.

- **(a)** Estimate the answer to the problem. Compare this estimate to the calculated value. Is your answer reasonable?
- **(b)** Repeat your calculations.
- **(c)** Work backward from your answer. Do you arrive at the original conditions of the problem?

EXAMPLE

A wire is $95\frac{1}{3}$ metres long. It is cut up into smaller, equal-sized pieces, each $4\frac{1}{3}$ metres long. How many pieces will there be?

1. ***Understand the problem.***

Draw a picture of the situation.

How will we find the number of pieces?

Now we will use a simpler problem to clarify the idea. A wire 100 metres long is cut up into smaller pieces each 4 metres long. How many pieces will there be? We readily see that we would divide 100 by 4. Thus in our original problem we should divide $95\frac{1}{3}$ metres by $4\frac{1}{3}$ metres. This will tell us the number of pieces. Now we fill in the Mathematics Blueprint (see below).

2. ***Solve and state the answer.***

We need to divide $95\frac{1}{3} \div 4\frac{1}{3}$.

$$\frac{286}{3} \div \frac{13}{3} = \frac{\overset{22}{\cancel{286}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{13}}} = \frac{22}{1} = 22$$

There will be 22 pieces of wire.

3. ***Check.***

Estimate. Rounded to the nearest ten, $95\frac{1}{3} \approx 100$.

Rounded to the nearest integer, $4\frac{1}{3} \approx 4$.

$$100 \div 4 = 25$$

This is close to our estimate. Our answer is reasonable. ✓

Mathematics Blueprint for Problem Solving

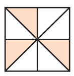
Gather the Facts	What Am I Asked to Do?	How Do I Proceed?	Key Points to Remember
Wire is $95\frac{1}{3}$ metres. It is cut into equal pieces $4\frac{1}{3}$ metres long.	Determine how many pieces of wire there will be.	Divide $95\frac{1}{3}$ by $4\frac{1}{3}$.	Change mixed numbers to improper fractions before carrying out the division.

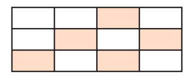
Module Review Problems

If you have trouble with a particular type of exercise, review the examples in the section indicated for that group of exercises. Answers to all exercises are located at the end of the module. Be sure to simplify all answers.

Section 1

Use a fraction to represent the shaded part of each object.

1. 

2. 

In exercises 3 and 4, draw a sketch to illustrate each fraction.

3. $\frac{4}{7}$ of an object

4. $\frac{7}{10}$ of a group

5. ***Quality Control*** An inspector looked at 80 semiconductors and found 9 of them defective. What fractional part of these items was defective?

6. ***Education*** The dean asked the 100 freshmen if they would be staying in the dorm over the holidays. A total of 87 said they would not. What fractional part of the freshmen said they would not?

Section 2

Express each number as a product of prime factors.

7. 54

8. 120

9. 168

Determine which of the following numbers are prime. If a number is composite, express it as the product of prime factors.

10. 59

11. 78

12. 167

Reduce each fraction.

13. $\frac{12}{42}$

14. $\frac{13}{52}$

15. $\frac{27}{72}$

16. $\frac{26}{34}$

17. $\frac{168}{192}$

18. $\frac{51}{105}$

Section 3

Change each mixed number to an improper fraction.

19. $4\frac{3}{8}$

20. $15\frac{3}{4}$

21. $5\frac{2}{7}$

22. $6\frac{3}{5}$

Change each improper fraction to a mixed number.

23. $\frac{45}{8}$

24. $\frac{100}{21}$

25. $\frac{53}{7}$

26. $\frac{74}{9}$

27. Reduce and leave your answer as a mixed number.
$3\frac{15}{55}$

28. Reduce and leave your answer as an improper fraction.
$\frac{234}{16}$

29. Change to a mixed number and then reduce.
$\frac{132}{32}$

Section 4

Multiply.

30. $\frac{4}{7} \times \frac{5}{11}$

31. $\frac{7}{9} \times \frac{21}{35}$

32. $12 \times \frac{3}{7} \times 0$

33. $\frac{3}{5} \times \frac{2}{7} \times \frac{10}{27}$

34. $12 \times 8\frac{1}{5}$

35. $5\frac{1}{4} \times 4\frac{6}{7}$

36. $5\frac{1}{8} \times 3\frac{1}{5}$

37. $36 \times \frac{4}{9}$

38. ***Stock Market*** In 1999, one share of stock cost $\$37\frac{5}{8}$. How much money did 18 shares cost?

▲ **39.** ***Geometry*** The O'Garas' new family room addition measures $4\frac{1}{2}$ metres long by $2\frac{2}{3}$ metres wide. Find the area of the addition.

Section 5

Divide, if possible.

40. $\frac{3}{7} \div \frac{2}{5}$

41. $\frac{3}{5} \div \frac{1}{10}$

42. $1200 \div \frac{5}{8}$

43. $900 \div \frac{3}{5}$

44. $5\frac{3}{4} \div 11\frac{1}{2}$

45. $\dfrac{20}{2\frac{1}{2}}$

46. $0 \div 3\frac{7}{5}$

47. $4\frac{2}{11} \div 3$

▲ **48.** ***Floor Carpeting*** Each roll of carpet covers $28\frac{1}{2}$ square metres. The community centre has 342 square metres of floor to carpet. How many rolls are needed?

49. There are 420 calories in $2\frac{1}{4}$ cans of grape pop. How many calories are in 1 can of pop?

Section 6

Find the LCD for each group of fractions.

50. $\frac{7}{14}$ and $\frac{3}{49}$

51. $\frac{13}{20}$ and $\frac{3}{25}$

52. $\frac{5}{18}, \frac{1}{6}, \frac{7}{45}$

Build each fraction to an equivalent fraction with the specified denominator.

53. $\frac{3}{7} = \frac{?}{56}$

54. $\frac{11}{24} = \frac{?}{72}$

55. $\frac{8}{15} = \frac{?}{150}$

56. $\frac{17}{18} = \frac{?}{198}$

Section 7

Add or subtract.

57. $\frac{9}{14} - \frac{5}{14}$

58. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

59. $\frac{4}{7} + \frac{7}{9}$

60. $\frac{7}{8} - \frac{3}{5}$

61. $\frac{7}{30} + \frac{2}{21}$

62. $\frac{5}{18} + \frac{7}{10}$

63. $\frac{15}{16} - \frac{13}{24}$

64. $\frac{14}{15} - \frac{3}{25}$

Section 8

Evaluate using the correct order of operations.

65. $8 - 2\frac{3}{4}$

66. $6 - \frac{5}{9}$

67. $3 + 5\frac{2}{3}$

68. $9\frac{3}{7} + 13$

69. $3\frac{3}{8} + 2\frac{3}{4}$

70. $5\frac{11}{16} - 2\frac{1}{5}$

71. $\frac{3}{5} \times \frac{1}{2} + \frac{2}{5} \div \frac{2}{3}$

72. $\left(\frac{4}{5} - \frac{1}{2}\right)^2 \times \frac{10}{3}$

73. ***Jogging*** Harminder jogged $1\frac{7}{8}$ kilometres on Monday, $2\frac{3}{4}$ kilometres on Tuesday, and $4\frac{1}{10}$ kilometres on Wednesday. How many kilometres did he jog on these three days?

74. ***Fuel Economy*** When it was new, Mariko's car got $18\frac{1}{6}$ kilometres per litre. It now gets $1\frac{5}{6}$ kilometres per litre less. How far can she drive now if the car has $10\frac{3}{4}$ litres in the tank?

Section 9

75. ***Cooking*** A recipe calls for $3\frac{1}{3}$ cups of sugar and $4\frac{1}{4}$ cups of flour. How much sugar and how much flour would be needed for $\frac{1}{2}$ of that recipe?

76. ***Fuel Economy*** Rafael travels in a car that gets $20\frac{1}{4}$ kilometres per litre. He has $8\frac{1}{2}$ litres of gas in the gas tank. Approximately how far can he drive?

77. ***Construction*** How many lengths of pipe $3\frac{1}{5}$ centimetres long can be cut from a pipe 48 centimetres long?

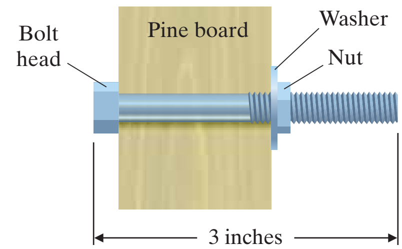
78. ***Automobile Maintenance*** A car radiator holds $15\frac{3}{4}$ litres. If it contains $6\frac{1}{8}$ litres of antifreeze and the rest is water, how much is water?

79. ***Reading Speed*** Tim found that he can read 5 pages of his biology book in $32\frac{1}{2}$ minutes. He has three chapters to read over the weekend. The first is 12 pages, the second is 9 pages, and the third is 14 pages. How long will it take him?

80. ***Personal Finance*** Tatiana earns $\$9\frac{1}{2}$ per hour for regular pay and $1\frac{1}{2}$ times that rate of pay for overtime. On Saturday she worked eight hours at regular pay and four hours at overtime pay. How much did she earn on Saturday?

81. ***Stock Market*** George bought 70 shares of stock in 2001 at $\$15\frac{3}{4}$ a share. He sold all the shares in 2003 for \$24 each. How much did George make when he sold his shares?

82. ***Carpentry*** A 3-inch bolt passes through $1\frac{1}{2}$ inches of pine board, a $\frac{1}{16}$-inch washer, and a $\frac{1}{8}$-inch nut. How many inches does the bolt extend beyond the board, washer, and nut if the head of the bolt is $\frac{1}{4}$ inch long?



83. ***Budgeting*** Francine has a take-home pay of \$880 per month. She gives $\frac{1}{10}$ of it to her church, spends $\frac{1}{2}$ of it for rent and food, and spends $\frac{1}{8}$ of it on electricity, heat, and telephone. How many dollars per month does she have left for other things?

84. ***Cost of Auto Travel*** Manuel's new car used $18\frac{2}{5}$ litres of gas on a 368-kilometre trip.

(a) How many kilometres can his car travel on 1 litre of gas?

(b) How much did his trip cost him in gasoline expense if the average cost of gasoline was $\$1\frac{1}{5}$ per litre?

Mixed Practice

Perform each calculation or each requested operation.

85. Reduce. $\dfrac{27}{63}$

86. $\dfrac{7}{15} + \dfrac{11}{25}$

87. $4\dfrac{1}{3} - 2\dfrac{11}{12}$

88. $\dfrac{36}{49} \times \dfrac{14}{33}$

89. $4\dfrac{1}{4} \div \dfrac{3}{2}$

90. $\left(\dfrac{4}{7}\right)^3$

91. $\dfrac{3}{8} \div \dfrac{1}{10}$

92. $5\dfrac{1}{2} \times 18$

93. $150 \div 3\dfrac{1}{8}$

1. ______

2. ______

3. ______

4. ______

5. ______

6. ______

7. ______

8. ______

9. ______

10. ______

11. ______

12. ______

13. ______

14. ______

15. ______

16. ______

17. ______

18. ______

How Am I Doing? Module Test

Solve.

1. Use a fraction to represent the shaded part of the object.

2. A basketball star shot at the hoop 388 times. The ball went in 311 times. Write a fraction that describes the part of the time that his shots went in.

Reduce each fraction.

3. $\frac{18}{42}$ **4.** $\frac{15}{70}$ **5.** $\frac{225}{50}$

6. Change to an improper fraction. $6\frac{4}{5}$

7. Change to a mixed number. $\frac{145}{14}$

Multiply.

8. $42 \times \frac{2}{7}$ **9.** $\frac{7}{9} \times \frac{2}{5}$ **10.** $2\frac{2}{3} \times 5\frac{1}{4}$

Divide.

11. $\frac{7}{8} \div \frac{5}{11}$ **12.** $\frac{12}{31} \div \frac{8}{13}$

13. $7\frac{1}{5} \div 1\frac{1}{25}$ **14.** $5\frac{1}{7} \div 3$

Find the least common denominator of each set of fractions.

15. $\frac{5}{12}$ and $\frac{7}{18}$ **16.** $\frac{3}{16}$ and $\frac{1}{24}$ **17.** $\frac{1}{4}, \frac{3}{8}, \frac{5}{6}$

18. Build the fraction to an equivalent fraction with the specified denominator. $\frac{5}{12} = \frac{?}{72}$

Evaluate using the correct order of operations.

19. $\frac{7}{9} - \frac{5}{12}$

20. $\frac{2}{15} + \frac{5}{12}$

21. $\frac{1}{4} + \frac{3}{7} + \frac{3}{14}$

22. $8\frac{3}{5} + 5\frac{4}{7}$

23. $18\frac{6}{7} - 13\frac{13}{14}$

24. $\frac{2}{9} \div \frac{8}{3} \times \frac{1}{4}$

25. $\left(\frac{1}{2} + \frac{1}{3}\right) \times \frac{7}{5}$

Answer each question.

▲ **26.** Erin needs to find the area of her kitchen so she knows how much tile to purchase. The room measures $16\frac{1}{2}$ feet by $9\frac{1}{3}$ feet. How many square feet is the kitchen?

27. A butcher has $18\frac{2}{3}$ kilograms of steak that he wishes to place into packages that average $2\frac{1}{3}$ kilograms each. How many packages can he make?

28. From central parking it is $\frac{9}{10}$ of a kilometre to the science building. Bob started at central parking and walked $\frac{1}{5}$ of a kilometre toward the science building. He stopped for coffee. When he finished, how much farther did he have to walk to reach the science building?

29. Robin jogged $4\frac{1}{8}$ kilometres on Monday, $3\frac{1}{6}$ kilometres on Tuesday, and $6\frac{3}{4}$ kilometres on Wednesday. How far did she jog on those three days?

30. Mr. and Mrs. Samuel visited Florida and purchased 120 oranges. They gave $\frac{1}{4}$ of them to relatives, ate $\frac{1}{12}$ of them in the hotel, and gave $\frac{1}{3}$ of them to friends. They shipped the rest home to Ontario.
(a) How many oranges did they ship?
(b) If it costs 24¢ for each orange to be shipped to Ontario, what was the total shipping bill?

31. A candle company purchased $48\frac{1}{8}$ kilograms of wax to make specialty candles. It takes $\frac{5}{8}$ kilogram of wax to make one candle. The owners of the business plan to sell the candles for $12 each. The specialty wax cost them $2 per kilogram.
(a) How many candles can they make?
(b) How much does it cost to make one candle?
(c) How much profit will they make if they sell all of the candles?

19. ____
20. ____
21. ____
22. ____
23. ____
24. ____
25. ____
26. ____
27. ____
28. ____
29. ____
30. (a) ____
(b) ____
31. (a) ____
(b) ____
(c) ____

Solutions to Practice Problems

Section 1 Practice Problems

1. (a) Four parts of twelve are shaded. The fraction is $\frac{4}{12}$.

(b) Three parts out of six are shaded. The fraction is $\frac{3}{6}$.

(c) Two parts of three are shaded. The fraction is $\frac{2}{3}$.

2. (a) Shade $\frac{4}{5}$ of the object. 

(b) Shade $\frac{3}{7}$ of the group. 

3. (a) $\frac{9}{17}$ represents 9 players out of 17.

(b) The total class is $382 + 351 = 733$.

The fractional part that is men is $\frac{382}{733}$.

(c) $\frac{7}{8}$ of a metre of material.

4. Total number of defective items $1 + 2 = 3$. Total number of items $7 + 9 = 16$. A fraction that represents the portion of the items that were defective is $\frac{3}{16}$.

Section 2 Practice Problems

1. (a)
$$\begin{aligned} 18 &= 2 \times 9 \\ &= 2 \times 3 \times 3 \\ &= 2 \times 3^2 \end{aligned}$$

(b)
$$\begin{aligned} 72 &= 8 \times 9 \\ &= 2 \times 2 \times 2 \times 3 \times 3 \\ &= 2^3 \times 3^2 \end{aligned}$$

(c)
$$\begin{aligned} 400 &= 10 \times 40 \\ &= 5 \times 2 \times 5 \times 8 \\ &= 5 \times 2 \times 5 \times 2 \times 2 \times 2 \\ &= 2^4 \times 5^2 \end{aligned}$$

2. (a) $\frac{30}{42} = \frac{30 \div 6}{42 \div 6} = \frac{5}{7}$

(b) $\frac{60}{132} = \frac{60 \div 12}{132 \div 12} = \frac{5}{11}$

3. (a) $\frac{120}{135} = \frac{2 \times 2 \times 2 \times \cancel{3} \times \cancel{5}}{3 \times 3 \times \cancel{3} \times \cancel{5}} = \frac{8}{9}$

(b) $\frac{715}{880} = \frac{\cancel{5} \times \cancel{11} \times 13}{2 \times 2 \times 2 \times 2 \times \cancel{5} \times \cancel{11}} = \frac{13}{16}$

4. (a) $\frac{84}{108} \stackrel{?}{=} \frac{7}{9}$

$84 \times 9 \stackrel{?}{=} 108 \times 7$

$756 = 756$ Yes

(b) $\frac{3}{7} \stackrel{?}{=} \frac{79}{182}$

$3 \times 182 \stackrel{?}{=} 7 \times 79$

$546 \neq 553$ No

Section 3 Practice Problems

1. (a) $4\frac{3}{7} = \frac{4 \times 7 + 3}{7} = \frac{28 + 3}{7} = \frac{31}{7}$

(b) $6\frac{2}{3} = \frac{6 \times 3 + 2}{3} = \frac{18 + 2}{3} = \frac{20}{3}$

(c) $19\frac{4}{7} = \frac{19 \times 7 + 4}{7} = \frac{133 + 4}{7} = \frac{137}{7}$

2. (a) $\begin{array}{r} 4 \\ 4\overline{)17} \\ \underline{16} \\ 1 \end{array}$ so $\frac{17}{4} = 4\frac{1}{4}$

(b) $\begin{array}{r} 7 \\ 5\overline{)36} \\ \underline{35} \\ 1 \end{array}$ so $\frac{36}{5} = 7\frac{1}{5}$

(c) $\begin{array}{r} 4 \\ 27\overline{)116} \\ \underline{108} \\ 8 \end{array}$ so $\frac{116}{27} = 4\frac{8}{27}$

(d) $\begin{array}{r} 7 \\ 13\overline{)91} \\ \underline{91} \\ 0 \end{array}$ so $\frac{91}{13} = 7$

3. $\frac{51}{15} = \frac{\overset{1}{\cancel{3}} \times 17}{\underset{1}{\cancel{3}} \times 5} = \frac{17}{5}$

4. $\frac{16}{80} = \frac{1}{5}$ so

$3\frac{16}{80} = 3\frac{1}{5}$.

5. $\frac{1001}{572} = 1\frac{429}{572}$

Now the fraction $\frac{429}{572} = \frac{3 \times \overset{1}{\cancel{11}} \times \overset{1}{\cancel{13}}}{2 \times 2 \times \underset{1}{\cancel{11}} \times \underset{1}{\cancel{13}}} = \frac{3}{4}$.

Thus $\frac{1001}{572} = 1\frac{429}{572} = 1\frac{3}{4}$.

Section 4 Practice Problems

1. (a) $\frac{6}{7} \times \frac{3}{13} = \frac{6 \times 3}{7 \times 13} = \frac{18}{91}$

(b) $\frac{1}{5} \times \frac{11}{12} = \frac{1 \times 11}{5 \times 12} = \frac{11}{60}$

2.
$$\begin{aligned} \frac{55}{72} \times \frac{16}{33} &= \frac{5 \cdot 11}{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3} \times \frac{2 \cdot 2 \cdot 2 \cdot 2}{3 \cdot 11} \\ &= \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 5 \cdot \cancel{11}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 3 \cdot 3 \cdot 3 \cdot \cancel{11}} \\ &= \frac{10}{27} \end{aligned}$$

3. (a) $7 \times \frac{5}{13} = \frac{7}{1} \times \frac{5}{13} = \frac{35}{13}$ or $2\frac{9}{13}$

(b) $\frac{13}{4} \times 8 = \frac{13}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{1} = \frac{26}{1} = 26$

4. $\frac{3}{\underset{1}{\cancel{8}}} \times \overset{12\,300}{\cancel{98\,400}} = \frac{3}{1} \times 12\,300 = 36\,900$

There are 36 900 square metres in the wetland area.

5. (a) $2\frac{1}{6} \times \frac{4}{7} = \frac{13}{\underset{3}{\cancel{6}}} \times \frac{\overset{2}{\cancel{4}}}{7} = \frac{26}{21}$ or $1\frac{5}{21}$

(b) $10\frac{2}{3} \times 13\frac{1}{2} = \frac{\overset{16}{\cancel{32}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{9}{\cancel{27}}}{\underset{1}{\cancel{2}}} = \frac{144}{1} = 144$

(c) $\frac{3}{5} \times 1\frac{1}{3} \times \frac{5}{8} = \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{4}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{8}}} = \frac{1}{2}$

(d) $3\frac{1}{5} \times 2\frac{1}{2} = \frac{\overset{8}{\cancel{16}}}{\underset{1}{\cancel{5}}} \times \frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}} = \frac{8}{1} = 8$

6. Area $= 1\frac{1}{5} \times 4\frac{5}{6} = \frac{\overset{1}{\cancel{6}}}{5} \times \frac{29}{\underset{1}{\cancel{6}}} = \frac{29}{5} = 5\frac{4}{5}$

The area is $5\frac{4}{5}$ square metres.

7. Since $8 \cdot 10 = 80$ and $9 \cdot 9 = 81$,

we know that $\frac{8}{9} \cdot \frac{10}{9} = \frac{80}{81}$.

Therefore $x = \frac{10}{9}$.

Section 5 Practice Problems

1. (a) $\frac{7}{13} \div \frac{3}{4} = \frac{7}{13} \times \frac{4}{3} = \frac{28}{39}$

(b) $\frac{16}{35} \div \frac{24}{25} = \frac{\overset{2}{\cancel{16}}}{\underset{7}{\cancel{35}}} \times \frac{\overset{5}{\cancel{25}}}{\underset{3}{\cancel{24}}} = \frac{10}{21}$

2. (a) $\frac{3}{17} \div 6 = \frac{3}{17} \div \frac{6}{1} = \frac{\overset{1}{\cancel{3}}}{17} \times \frac{1}{\underset{2}{\cancel{6}}} = \frac{1}{34}$

(b) $14 \div \frac{7}{15} = \frac{14}{1} \div \frac{7}{15} = \frac{\overset{2}{\cancel{14}}}{1} \times \frac{15}{\underset{1}{\cancel{7}}} = 30$

3. (a) $1 \div \frac{11}{13} = \frac{1}{1} \times \frac{13}{11} = \frac{13}{11}$ or $1\frac{2}{11}$

(b) $\frac{14}{17} \div 1 = \frac{14}{17} \times \frac{1}{1} = \frac{14}{17}$

(c) $\frac{3}{11} \div 0$ Division by zero is undefined.

(d) $0 \div \frac{9}{16} = \frac{0}{1} \times \frac{16}{9} = \frac{0}{9} = 0$

4. (a) $1\frac{1}{5} \div \frac{7}{10} = \frac{6}{5} \div \frac{7}{10} = \frac{6}{\underset{1}{\cancel{5}}} \times \frac{\overset{2}{\cancel{10}}}{7} = \frac{12}{7}$ or $1\frac{5}{7}$

(b) $2\frac{1}{4} \div 1\frac{7}{8} = \frac{9}{4} \div \frac{15}{8} = \frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{5}{\cancel{15}}} = \frac{6}{5}$ or $1\frac{1}{5}$

5. (a) $\frac{5\frac{2}{3}}{7} = 5\frac{2}{3} \div 7 = \frac{17}{3} \times \frac{1}{7} = \frac{17}{21}$

(b) $\frac{1\frac{2}{5}}{2\frac{1}{3}} = 1\frac{2}{5} \div 2\frac{1}{3} = \frac{7}{5} \div \frac{7}{3} = \frac{\overset{1}{\cancel{7}}}{5} \times \frac{3}{\underset{1}{\cancel{7}}} = \frac{3}{5}$

6. $x \div \frac{3}{2} = \frac{22}{36}$

$x \cdot \frac{2}{3} = \frac{22}{36}$

$\frac{11}{12} \cdot \frac{2}{3} = \frac{22}{36}$ Thus $x = \frac{11}{12}$.

7. $19\frac{1}{4} \div 14 = \frac{\overset{11}{\cancel{77}}}{4} \times \frac{1}{\underset{2}{\cancel{14}}} = \frac{11}{8}$ or $1\frac{3}{8}$

Each piece will be $1\frac{3}{8}$ metres long.

Section 6 Practice Problems

1. The multiples of 14 are 14, 28, 42, 56, 70, 84, ...
The multiples of 21 are 21, 42, 63, 84, 105, 126, ...
42 is the least common multiple of 14 and 21.

2. The multiples of 10 are 10, 20, 30, 40 ...
The multiples of 15 are 15, 30, 45 ...
30 is the least common multiple of 10 and 15.

3. 54 is a multiple of 6. We know that $6 \times 9 = 54$.
The least common multiple of 6 and 54 is 54.

4. (a) The LCD of $\frac{3}{4}$ and $\frac{11}{12}$ is 12.

12 can be divided by 4 and 12.

(b) The LCD of $\frac{1}{7}$ and $\frac{8}{35}$ is 35.

35 can be divided by 7 and 35.

5. The LCD of $\frac{3}{7}$ and $\frac{5}{6}$ is 42.

42 can be divided by 7 and 6.

6. (a) $14 = 2 \times 7$
$10 = 2 \times 5$
$\text{LCD} = 2 \times 5 \times 7 = 70$

(b) $15 = 3 \times 5$
$50 = 2 \times 5 \times 5$
$\text{LCD} = 2 \times 3 \times 5 \times 5 = 150$

(c) $16 = 2 \times 2 \times 2 \times 2$
$12 = 2 \times 2 \times 3$
$\text{LCD} = 2 \times 2 \times 2 \times 2 \times 3 = 48$

7. $49 = 7 \times 7$
$21 = 7 \times 3$
$7 = 7 \times 1$
$\text{LCD} = 7 \times 7 \times 3 = 147$

8. (a) $\frac{3}{5} = \frac{3}{5} \times \frac{8}{8} = \frac{24}{40}$

(b) $\frac{7}{11} = \frac{7}{11} \times \frac{4}{4} = \frac{28}{44}$

(c) $\frac{2}{7} = \frac{2}{7} \times \frac{4}{4} = \frac{8}{28}$

$\frac{3}{4} = \frac{3}{4} \times \frac{7}{7} = \frac{21}{28}$

9. (a) $20 = 2 \times 2 \times 5$
$15 = 3 \times 5$
$\text{LCD} = 2 \times 2 \times 3 \times 5 = 60$

(b) $\frac{3}{20} = \frac{3}{20} \times \frac{3}{3} = \frac{9}{60}$ $\quad \frac{11}{15} = \frac{11}{15} \times \frac{4}{4} = \frac{44}{60}$

10. (a) $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$80 = 2 \times 2 \times 2 \times 2 \times 5$
$\text{LCD} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 = 320$

(b) $\frac{5}{64} = \frac{5}{64} \times \frac{5}{5} = \frac{25}{320}$

$\frac{3}{80} = \frac{3}{80} \times \frac{4}{4} = \frac{12}{320}$

Section 7 Practice Problems

1. $\frac{3}{17} + \frac{12}{17} = \frac{15}{17}$

2. (a) $\frac{1}{12} + \frac{5}{12} = \frac{6}{12} = \frac{1}{2}$

(b) $\frac{13}{15} + \frac{7}{15} = \frac{20}{15} = \frac{4}{3}$ or $1\frac{1}{3}$

3. (a) $\frac{5}{19} - \frac{2}{19} = \frac{3}{19}$ **(b)** $\frac{21}{25} - \frac{6}{25} = \frac{15}{25} = \frac{3}{5}$

4.
$$\begin{array}{rcr} \frac{2}{15} & = & \frac{2}{15} \\ +\frac{1}{5} \times \frac{3}{3} & = & +\frac{3}{15} \\ \hline & & \frac{5}{15} = \frac{1}{3} \end{array}$$

5. $\text{LCD} = 48$ $\quad \frac{5}{12} \times \frac{4}{4} = \frac{20}{48}$ $\quad \frac{5}{16} \times \frac{3}{3} = \frac{15}{48}$

$\frac{5}{12} + \frac{5}{16} = \frac{20}{48} + \frac{15}{48} = \frac{35}{48}$

6. $\text{LCD} = 48$

$\frac{3}{16} \times \frac{3}{3} = \frac{9}{48}$ $\quad \frac{1}{8} \times \frac{6}{6} = \frac{6}{48}$ $\quad \frac{1}{12} \times \frac{4}{4} = \frac{4}{48}$

$\frac{3}{16} + \frac{1}{8} + \frac{1}{12} = \frac{9}{48} + \frac{6}{48} + \frac{4}{48} = \frac{19}{48}$

7. $\text{LCD} = 96$ $\quad \frac{9}{48} \times \frac{2}{2} = \frac{18}{96}$ $\quad \frac{5}{32} \times \frac{3}{3} = \frac{15}{96}$

$\frac{9}{48} - \frac{5}{32} = \frac{18}{96} - \frac{15}{96} = \frac{3}{96} = \frac{1}{32}$

8. $\frac{9}{10} \times \frac{2}{2} = \frac{18}{20}$ $\quad \frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$

$\frac{9}{10} - \frac{1}{4} = \frac{18}{20} - \frac{5}{20} = \frac{13}{20}$

There is $\frac{13}{20}$ litre left.

9. The LCD of $\frac{3}{10}$ and $\frac{23}{25}$ is 50.

$\frac{3}{10} \times \frac{5}{5} = \frac{15}{50}$ Now rewriting: $x + \frac{15}{50} = \frac{46}{50}$

$\frac{23}{25} \times \frac{2}{2} = \frac{46}{50}$ $\frac{31}{50} + \frac{15}{50} = \frac{46}{50}$

So, $x = \frac{31}{50}$

10. $\frac{15}{16} + \frac{3}{40}$

$\frac{15}{16} \times \frac{40}{40} = \frac{600}{640}$ $\frac{3}{40} \times \frac{16}{16} = \frac{48}{640}$

Thus $\frac{15}{16} + \frac{3}{40} = \frac{600}{640} + \frac{48}{640} = \frac{648}{640} = \frac{81}{80}$ or $1\frac{1}{80}$

Section 8 Practice Problems

1.
$$\begin{array}{r} 5\frac{1}{12} \\ +\,9\frac{5}{12} \\ \hline 14\frac{6}{12} = 14\frac{1}{2} \end{array}$$

2. The LCD is 20.

$\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}$ $\frac{2}{5} \times \frac{4}{4} = \frac{8}{20}$

$$\begin{array}{rcr} 6\frac{1}{4} & = & 6\frac{5}{20} \\ +\,2\frac{2}{5} & = & +2\frac{8}{20} \\ \hline & & 8\frac{13}{20} \end{array}$$

3. LCD = 12
$$\begin{array}{rcr} 7\boxed{\frac{1}{4} \times \frac{3}{3}} & = & 7\frac{3}{12} \\ +3\boxed{\frac{5}{6} \times \frac{2}{2}} & = & +3\frac{10}{12} \\ \hline & & 10\frac{13}{12} = 10 + 1\frac{1}{12} = 11\frac{1}{12} \end{array}$$

4. LCD = 12
$$\begin{array}{rcr} 12\frac{5}{6} & = & 12\frac{10}{12} \\ -\,7\frac{5}{12} & = & -7\frac{5}{12} \\ \hline & & 5\frac{5}{12} \end{array}$$

5. (a) LCD = 24
$$\begin{array}{rcrcr} 9\boxed{\frac{1}{8} \times \frac{3}{3}} & = & 9\frac{3}{24} & = & 8\frac{27}{24} \\ -3\boxed{\frac{2}{3} \times \frac{8}{8}} & = & -3\frac{16}{24} & = & -3\frac{16}{24} \\ \hline & & & & 5\frac{11}{24} \end{array}$$

Borrow 1 from 9:

$9\frac{3}{24} = 8 + 1\frac{3}{24} = 8\frac{27}{24}$

(b)
$$\begin{array}{rcr} 18 & = & 17\frac{18}{18} \\ -6\frac{7}{18} & = & -6\frac{7}{18} \\ \hline & & 11\frac{11}{18} \end{array}$$

6.
$$\begin{array}{rcrcr} 6\frac{1}{4} & = & 6\frac{3}{12} & = & 5\frac{15}{12} \\ -4\frac{2}{3} & = & -4\frac{8}{12} & = & -4\frac{8}{12} \\ \hline & & & & 1\frac{7}{12} \end{array}$$

They had $1\frac{7}{12}$ litres left over.

7. $\frac{3}{5} - \frac{1}{15} \times \frac{10}{13}$

$= \frac{3}{5} - \frac{2}{39}$ LCD $= 5 \cdot 39 = 195$

$= \frac{117}{195} - \frac{10}{195}$

$= \frac{107}{195}$

8. $\frac{1}{7} \times \frac{5}{6} + \frac{5}{3} \div \frac{7}{6} = \frac{1}{7} \times \frac{5}{6} + \frac{5}{3} \times \frac{6}{7}$

$= \frac{5}{42} + \frac{10}{7}$ LCD = 42

$= \frac{5}{42} + \frac{60}{42}$

$= \frac{65}{42}$ or $1\frac{23}{42}$

Section 9 Practice Problems

Practice Problem 1

1. Understand the problem.

Mathematics Blueprint for Problem Solving

Gather the Facts	What Am I Asked to Do?	How Do I Proceed?	Key Points to Remember
Gas amounts: $18\frac{7}{10}$ L, $15\frac{2}{5}$ L, $14\frac{1}{2}$ L	Find out how many litres of gas she bought altogether.	Add the three amounts.	When adding mixed numbers, the LCD is needed for the fractions.

2. Solve and state the answer:

LCD = 10

$$\begin{aligned} 18\tfrac{7}{10} &= 18\tfrac{7}{10} \\ 15\tfrac{2}{5} &= 15\tfrac{4}{10} \\ 14\tfrac{1}{2} &= \underline{+14\tfrac{5}{10}} \\ & 47\tfrac{16}{10} = 48\tfrac{6}{10} \\ & \phantom{= 47\tfrac{16}{10}} = 48\tfrac{3}{5} \end{aligned}$$

The total is $48\frac{3}{5}$ litres.

3. ***Check.*** Estimate to see if the answer is reasonable.

Practice Problem 2

1. Understand the problem.

Mathematics Blueprint for Problem Solving			
Gather the Facts	**What Am I Asked to Do?**	**How Do I Proceed?**	**Key Points to Remember**
Postcard: $12\frac{1}{4}$ cm Top border: $1\frac{3}{8}$ cm Bottom border: 2 cm	Find the length of the inside portion of the postcard.	**(a)** Add the two border lengths. **(b)** Subtract this total from the postcard length.	When adding mixed numbers, the LCD is needed for the fractions.

2. Solve and state the answer:

(a) $$\begin{array}{r} 1\frac{3}{8} \\ \underline{+2\phantom{\frac{3}{8}}} \\ 3\frac{3}{8} \end{array}$$

(b) $$12\frac{1}{4} = 12\frac{2}{8} = 11\frac{10}{8}$$
$$-3\frac{3}{8} = -3\frac{3}{8} = \underline{-3\frac{3}{8}}$$
$$8\frac{7}{8}$$

The length of the inside portion is $8\frac{7}{8}$ centimetres.

3. ***Check.*** Estimate to see if the answer is reasonable or work backward to check.

Practice Problem 3

1. Understand the problem.

Mathematics Blueprint for Problem Solving			
Gather the Facts	**What Am I Asked to Do?**	**How Do I Proceed?**	**Key Points to Remember**
Regular tent uses $8\frac{1}{4}$ metres. Large tent uses $1\frac{1}{2}$ times the regular. She makes 6 regular and 16 large tents.	Find out how many metres of cloth will be needed to make the tents.	Find the amount used for regular tents, and the amount used for large tents. Then add the two.	Large tents use $1\frac{1}{2}$ times the regular amount.

2. Solve and state the answer:

We multiply $6 \times 8\frac{1}{4}$ for regular tents and $16 \times 1\frac{1}{2} \times 8\frac{1}{4}$ for large tents. Then add total metrage.

Regular tents: $6 \times 8\frac{1}{4} = \frac{\cancelto{3}{6}}{} \times \frac{33}{\cancelto{2}{4}} = \frac{99}{2} = 49\frac{1}{2}$

Large tents: $16 \times 1\frac{1}{2} \times 8\frac{1}{4} = \cancelto{2}{\cancelto{8}{16}} \times \frac{3}{\cancelto{1}{2}} \times \frac{33}{\cancelto{1}{4}} = \frac{198}{1} = 198$

Total metrage for all tents is $198 + 49\frac{1}{2} = 247\frac{1}{2}$ metres.

3. ***Check.*** Estimate to see if the answer is reasonable.

Practice Problem 4

1. Understand the problem.

Mathematics Blueprint for Problem Solving

Gather the Facts	What Am I Asked to Do?	How Do I Proceed?	Key Points to Remember
He purchases 4-metre boards. Each shelf is $1\frac{1}{4}$ m. He needs four shelves for each bookcase and he is making two bookcases.	**(a)** Find out how many boards he needs to buy. **(b)** Find out how many metres of shelving are actually needed. **(c)** Find out how many metres will be left over.	Find out how many $1\frac{1}{4}$-m shelves he can get from one board. Then see how many boards he needs to make all eight shelves.	There will be three answers to this problem. Don't forget to calculate the leftover wood.

2. Solve and state the answer:

We want to know how many $1\frac{1}{4}$-m shelves are in a 4-m board.

$$4 \div 1\frac{1}{4} = \frac{4}{1} \div \frac{5}{4} = \frac{4}{1} \times \frac{4}{5} = \frac{16}{5} = 3\frac{1}{5}$$

He will get 3 shelves from each board with some left over.

(a) For two bookcases, he needs eight shelves. He gets three shelves out of each board. $8 \div 3 = 2.67 \approx 3$. He will need three 4-m boards.

(b) He needs 8 shelves at $1\frac{1}{4}$ m.

$$8 \times 1\frac{1}{4} = 8 \times \frac{5}{4} = 10$$

He actually needs 10 metres of shelving.

(c)
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   12 metres of shelving bought
 − 10 metres of shelving used
    2 metres of shelving left over.
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3. ***Check.*** Work backward to check the answer.

Practice Problem 5

1. Understand the problem.

Mathematics Blueprint for Problem Solving

Gather the Facts	What Am I Asked to Do?	How Do I Proceed?	Key Points to Remember
Distance is $199\frac{3}{4}$ kilometres. He uses $8\frac{1}{2}$ litres of gas.	Find out how many kilometres per litre he gets.	Divide the distance by the number of litres.	Change mixed numbers to improper fractions before dividing.

2. Solve and state the answer:

$$199\frac{3}{4} \div 8\frac{1}{2} = \frac{799}{4} \div \frac{17}{2}$$

$$= \frac{\overset{47}{\cancel{799}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{17}}}$$

$$= \frac{47}{2} = 23\frac{1}{2}$$

He gets $23\frac{1}{2}$ kilometres per litre.

3. ***Check.*** Estimate to see if the answer is reasonable.

Glossary

Building fraction property (Section 6) For whole numbers a, b, and c, where neither b nor c equals zero,

$$\frac{a}{b} = \frac{a}{b} \times 1 = \frac{a}{b} \times \frac{c}{c} = \frac{a \times c}{b \times c}.$$

Building up a fraction (Section 6) To make one fraction into an equivalent fraction by making the denominator and numerator larger numbers. For example, the fraction $\frac{3}{4}$ can be built up to the fraction $\frac{30}{40}$.

Common denominator (Section 7) Two fractions have a common denominator if the same number appears in the denominator of each fraction. $\frac{3}{7}$ and $\frac{1}{7}$ have a common denominator of 7.

Composite number (Section 2) A composite number is a whole number greater than 1 that can be divided by whole numbers other than itself. The number 6 is a composite number since it can be divided exactly by 2 and 3 (as well as by 1 and 6).

Denominator (Section 1) The number on the bottom of a fraction. In the fraction $\frac{2}{9}$ the denominator is 9.

Equal fractions (Section 2) Fractions that represent the same number. The fractions $\frac{3}{4}$ and $\frac{6}{8}$ are equal fractions.

Equality test of fractions (Section 2) Two fractions $\frac{a}{b}$ and $\frac{c}{d}$ are equal if the product $a \times d = b \times c$. In this case, a, b, c, and d are whole numbers and b and $d \neq 0$.

Equivalent fractions (Section 2) Two fractions that are equal.

Fundamental theorem of arithmetic (Section 2) Every composite number has a unique product of prime numbers.

Improper fraction (Section 3) A fraction in which the numerator is greater than or equal to the denominator. The fractions $\frac{34}{29}$, $\frac{8}{7}$, and $\frac{6}{6}$ are all improper fractions.

Invert a fraction (Section 5) To invert a fraction is to interchange the numerator and the denominator. If we invert $\frac{5}{9}$, we obtain the fraction $\frac{9}{5}$. To invert a fraction is sometimes referred to as *to take the reciprocal of a fraction*.

Irreducible (Section 2) A fraction that cannot be reduced (simplified) is called irreducible.

Least common denominator (LCD) (Section 6) The least common denominator (LCD) of two or more fractions is the smallest number that can be divided without remainder by each fraction's denominator. The LCD of $\frac{1}{3}$ and $\frac{1}{4}$ is 12. The LCD of $\frac{5}{6}$ and $\frac{4}{15}$ is 30.

Mixed number (Section 3) A number created by the sum of a whole number greater than 1 and a proper fraction. The numbers $4\frac{5}{6}$ and $1\frac{1}{8}$ are both mixed numbers. Mixed numbers are sometimes referred to as *mixed fractions*.

Numerator (Section 1) The number on the top of a fraction. In the fraction $\frac{3}{7}$ the numerator is 3.

Overtime (Section 9) The pay earned by a person if he or she works more than a certain number of hours per week. In most jobs that pay by the hour, a person will earn $1\frac{1}{2}$ times as much per hour for every hour beyond 40 hours worked in one workweek. For example, Carlos earns $6.00 per hour for the first 40 hours in a week and overtime for each additional hour. He would earn $9.00 per hour for all hours he worked in that week beyond 40 hours.

Prime factors (Section 2) Factors that are prime numbers. If we write 15 as a product of prime factors, we have $15 = 5 \times 3$.

Prime number (Section 2) A prime number is a whole number greater than 1 that can only be divided by 1 and itself. The first fifteen prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47. The list of prime numbers goes on forever.

Proper fraction (Section 3) A fraction in which the numerator is less than the denominator. The fractions $\frac{3}{4}$ and $\frac{15}{16}$ are proper fractions.

Reduced fraction (Section 2) A fraction for which the numerator and denominator have no common factor other than 1. The fraction $\frac{5}{7}$ is a reduced fraction. The fraction $\frac{15}{21}$ is not a reduced fraction because both numerator and denominator have a common factor of 3.